We will see another way to write square (and other) roots.

Intermediate algebra
Class notes
Rational Exponents (section 17.2)

Rational exponents: Rational exponents are simply another way to write roots. For instance, instead of writing $\sqrt{x}$, we could write $x^{1/2}$. This makes it easier to work with radicals because we can use the rules of exponents we studied previously.

Likewise, we will write $x^{1/3}$ to mean $\sqrt[3]{x}$. In general, we will write $x^{1/n}$ to mean $\sqrt[n]{x}$.

expl 1: Write as a radical expression. Simplify if possible.

a.) $25^{1/2}$

b.) $(-8)^{1/3}$

c.) $\left(\frac{1}{16}\right)^{1/4}$

expl 2: Write as a radical expression. Assume all variables represent non-negative numbers. Simplify if possible.

a.) $\left(27x^3\right)^{1/3}$

Deal with each factor separately.

b.) $\left(81x^6y^2\right)^{1/2}$

$\sqrt{x^6} = x^3$ since $\left(x^3\right)^2 = x^6$

Using rules of exponents to simplify expressions: These problems are essentially the same as we have done before, except the exponents will now be fractions. Try to fill in the rules of exponents below from memory.

Product rule: $a^m \cdot a^n =$

$2^3 \cdot 2^4 = (2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2)$
How many 2's is that? So what is the product rule?

Quotient rule: $\dfrac{a^m}{a^n} =$

Power rule: $\left(a^m\right)^n =$

Power of a product rule: $(a \cdot b)^n =$

Power of a quotient rule: $\left(\dfrac{a}{c}\right)^n =$

Zero exponent rule: $a^0 =$ (Here *a* cannot be 0 because 0^0 is undefined.)

Negative exponent rule: $a^{-n} =$ and $\dfrac{1}{a^{-n}} =$ (if *a* is non-zero)

Negative rational exponents: The last rule says that $a^{-n} = \dfrac{1}{a^n}$. So, we now have $a^{-1/n} = \dfrac{1}{a^{1/n}}$.

This assumes $a^{1/n}$ is non-zero. Why?

expl 3a: Multiply.

$$\left(x^{1/3} + 5\right)\left(x^{1/3} - 5\right)$$

expl 3b: Factor $a^{1/3}$ out of the expression $a^{2/3} + 4a^{1/3}$.

expl 4: Simplify each expression. Your final answer should contain only positive exponents. Assume all variables represent non-negative numbers.

> You may need to review operating with fractions. Or just use the calculator.

a.) $x^{2/5} \cdot x^{1/4}$

b.) $\left(n^{3/7}\right)^{5/9}$

c.) $\dfrac{\left(y^3 t^2\right)^{1/2}}{(yt)^{-1/3}}$

d.) $\dfrac{2x}{x^{-1/4}}$

Rational exponents with numerators other than 1: If $x^{1/3}$ means $\sqrt[3]{x}$, then what does $x^{2/3}$ mean? If we write it as $\left(x^2\right)^{1/3}$, we see it is $\sqrt[3]{x^2}$.

> Flipped the order of exponents but it's equivalent!

But, funnily enough, you could also write $x^{2/3}$ as $\left(x^{1/3}\right)^2$. So, $x^{2/3}$ could also be thought of as $\left(\sqrt[3]{x}\right)^2$. Do you see the difference? In general, $a^{m/n} = \sqrt[n]{a^m}$ or $\left(\sqrt[n]{a}\right)^m$.

(This assumes *m* and *n* are positive integers, m/n is in lowest terms, and $\sqrt[n]{a}$ is a real number.)

Likewise, $a^{-m/n} = \dfrac{1}{a^{m/n}} = \dfrac{1}{\sqrt[n]{a^m}}$ or $\dfrac{1}{\left(\sqrt[n]{a}\right)^m}$.

If we need to find $64^{2/3}$ without the calculator, would you want to think of it as $\left(64^2\right)^{1/3}$ or $\left(64^{1/3}\right)^2$.

Recall $4^3 = 64$.

expl 5: Use radical notation to rewrite the expression. Simplify if possible.

a.) $x^{3/5}$

b.) $(-27)^{4/3}$

Think $\left(\sqrt[3]{-27}\right)^4$. Follow order of operations!

c.) $(-16)^{3/2}$

Think $\left(\sqrt{-16}\right)^3$. What goes wrong?

d.) $25^{-3/2}$

Think $\left(25^{1/2}\right)^{-3}$. Follow order of operations.

expl 6: Use rational exponents to simplify the radical expression. Assume that variables represent positive numbers. Once simplified, convert back to radical form if applicable.

a.) $\sqrt[6]{x^3}$

b.) $\sqrt[3]{w^6 t^9}$

c.) $\sqrt[8]{(x+3)^4}$

d.) $\sqrt[4]{t^3} \cdot \sqrt[5]{t^2}$

Calculator note: Square roots, cube roots: You will often want to know the cube or square root of a number.

On the TI-82, 83, and 84, press the MATH button and you will find two useful functions, $\sqrt[3]{\ }($ and $\sqrt[x]{\ }$. The first is the cube root function. The second will be used to find other roots, such as the fourth or tenth roots.

On the TI-86, enter the MATH menu by pressing the 2nd button and then the multiplication button. You will find the $\sqrt[x]{\ }$ under the MISC menu (once you are in the MATH menu, press F5 and then MORE.) This will find all roots, such as the cube or tenth root. There is no specific cube root function like the calculators above have.

We will use the $\sqrt[x]{\ }$ function because it is a root other than cube or square.

expl 7: Find the fourth root of 625. Enter the following. (Instructions are for the TI-82, 83, and 84 calculators.)

4 [MATH] [5] 625 [ENTER]

You must tell it which root (fourth) to find.

Selects $\sqrt[x]{\ }$

Check your answer by raising it to the fourth power to see if you get 625.

Also, how else could you write $\sqrt[4]{625}$ using rational exponents? You could enter that into the calculator, but make sure you put parentheses around the exponent. Try it now.