Recall we factor 10 as $5 \cdot 2$.
Factoring something means to think of it as a product!

Intermediate algebra
Class notes
Factoring Introduction (section 13.1)

Factors versus terms:

terms: things we are adding (or subtracting)
expls: $x + 4$ or $2x + 3$ or $4x^2 + 3x - 6$

factors: things we are multiplying (or dividing)
expls: $5 \cdot x$ or $3(x+2)$ or $4 \cdot x^2$

Could be thought of as $4 \cdot x \cdot x$ or $2 \cdot 2 \cdot x \cdot x$. What are the factors then?

Add your own examples to the lists above.

To factor an expression like $x^2 + 6x + 5$ means to **write it as a product** "something times something" **instead of a sum** "something plus something plus something". Do you see how $x^2 + 6x + 5$ is a sum of three terms? What are those terms?

Recall that to factor 20, we write it as a product, $4 \cdot 5$ or $2 \cdot 2 \cdot 5$. To factor $x^2 + 6x + 5$, we also write it as a product. We will learn how to write $x^2 + 6x + 5$ as $(x+5)(x+1)$. We can think of this as the product of two factors. What are those factors?

Check that I am not lying to you. FOIL out $(x+5)(x+1)$ to make sure it really is $x^2 + 6x + 5$.

Another example: The expression 3x + 12 can be thought of as a **sum** of the terms 3x and 12. But it is also equivalent to 3(x+4), which could be thought of as the **product** of two factors. What are they?

The fact that 3x + 12 = 3(x+4) is a lovely example of which property of real numbers?

a(b + c) = ab + ac

This will be used to factor expressions by drawing out the GCF (greatest common factor) from two (or more) terms like the 3 in this example.

To factor an expression:

1. Always start by factoring out the GCF from **all** terms if possible.

2. Various methods will be discussed later. If you are given a …

   2-term expression like $x^2 - 16$, look into special formulas,
   3-term expression like $2x^2 - 5x - 3$, try the various methods given for trinomials,
   4-term expression like $2x^2 - 6x + 1x - 3$, try factoring by grouping.

3. After you factor, check the individual factors for additional factoring possibilities.

4. Always check your final answer by multiplying it out. Remember an expression and its factored form should be equivalent.

5. Some expressions are **prime**; they cannot be factored.

Greatest Common Factor (GCF):

The GCF of two numbers is the largest number that is a factor of both original numbers.

expl 1: Find the GCF of 10 and 45.

What are the factors of 10 and 45?

10: 1, 2, 5, 10

45: 1, 3, 5, 9, 15, 45

Of the factors of 10 and 45, which is the largest common factor? That is the GCF.

expl 2: Find the GCF of $6x^2$ and $10x^3$.

Same thing here, but the x^2 and x^3 parts feel weird. What is the largest factor they both contain?

x^2 is $x \cdot x$

x^3 is $x \cdot x \cdot x$

expl 3: Find the GCF of $6x^2y^5z^3$, $9xy^2z$, and $30x^2y^2z^3$.

Recall the Distribution Property:

It allows us to write a sum as a product, which is what factoring is all about. We use the distribution property to factor 3x + 12 by pulling out the GCF of 3.

$ab + ac = a(b + c)$

$3x + 12 = 3(x+4)$

sum → $3x + 12$; product → $3(x+4)$

expl 4: Factor the GCF out of the expression $18a^3 - 27a^5 + 9a^4$.

What is the GCF of the three terms? Pull it out in front using the Distribution Property.

Is your final answer written as a product? Check your answer by multiplying it back out.

expl 5: Factor the GCF out of the expression $25r^3s^4 + 10r^2s^5 - 15r^4s$.

It sometimes helps to write all terms in expanded form. The term $25r^3s^4$ could be written as $5 \cdot 5 \cdot r \cdot r \cdot r \cdot s \cdot s \cdot s \cdot s$.

expl 6: Factor $4(x + 2) + x(x + 2)$.

Here, see you have two terms, double-underlined below.

$\underline{4(x + 2)} + \underline{x(x + 2)}$

What is the common factor? Factor it out like before.

expl 7: Factor $x(4x + 3) + 4(4x + 3)$.

Remember the point is to write it as a product. It is counterproductive to start by multiplying it all out.

Factor by grouping: We will use this method to factor 4-term expressions. It will also be used to help factor trinomials ("3-terms") in one of the various methods we will cover later.

expl 8: Factor by grouping $3x^2 + 15x + 4x + 20$.

Idea: Factor something out of first 2 terms. Factor something out of last 2 terms. What's left over in each should match and we proceed like last couple of examples.

$\underline{3x^2 + 15x} + \underline{4x + 20}$

What's common? What's common?

expl 9: Factor by grouping $x^2 – 8x – 5x + 40$.

Rewrite "minus 5x" in middle as "plus negative 5x" to make it easier.

$\underline{x^2 – 8x} + \underline{-5x + 40}$

What's common? What's common?

Remember the stuff left over in both cases should be the same. Can you work it out?

expl 10: Find GCF of $30x^3y^4z^5$, $12x^6y^{11}z^3$, and $24y^4z$.

expl 11: Factor by grouping $x^3 + 8x^2 + 6x + 48$.

expl 12: Factor by using the Distribution Property $14x^2y + 7xy – 21x$.

Worksheet: Graphing calculator basics (TI82, 83, 84, 85, 86):
This worksheet starts at the beginning and covers what you need to get started including but not limited to second and alpha functions and basic graphing for common TI calculators. Ignore paragraphs that reference calculators that are not your model.

Worksheet: Things to know about your calculator (Texas instruments – 82, 83, 85, 86):
This is a laundry list of stuff I have found useful over the years. You should go through the items one at a time, trying them out for yourself. Again, ignore paragraphs that reference calculators that are not your model. However, if you have a TI84, you should find the material for the TI83 should also work for you.