Intermediate algebra
Class notes
Factoring Binomials like $x^2 - 16$ using Formulas (section 13.5)

We will use formulas to factor certain binomials that are harder (but not impossible) to do by trial and error.

Formulas in section:

Difference of two squares:

Memorize this one. It will be used the most.

$a^2 - b^2 = (a + b)(a - b)$

Notice 16 is 4^2. So b here is 4.

expl: $x^2 - 16 = (x + 4)(x - 4)$

Difference of two cubes:

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Why 3t? Why 9?

expl: $t^3 - 27 = (t - 3)(t^2 + 3t + 9)$

The missing one, sum of two squares $a^2 + b^2$, cannot be factored. Never, no-how, no way.

Sum of two cubes:

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

expl: $t^3 + 27 = (t + 3)(t^2 - 3t + 9)$

It's never a good idea to take rules on faith. Let's verify the "difference of two cubes" rule for ourselves. Multiply $(a - b)(a^2 + ab + b^2)$ to see how it really does make $a^3 - b^3$.

expl 1: Factor completely.

$x^2 - 9$

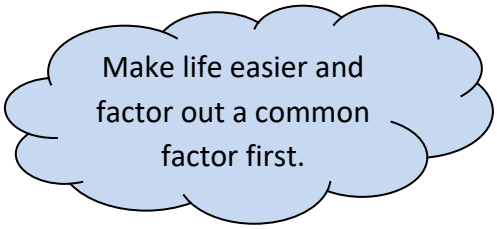
What's in front, stays in front.

expl 2: Factor completely.

$144 - x^2$

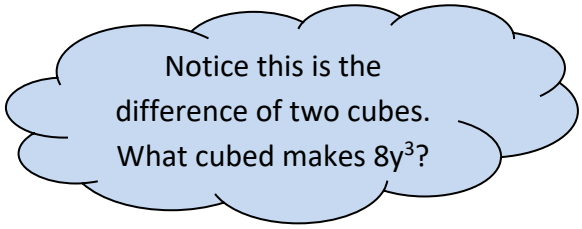
expl 3: Factor completely.

$8x^2 - 392$



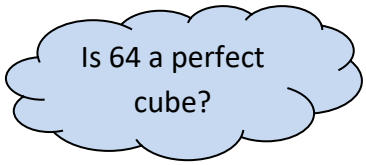
expl 4: Factor completely.

$x^3 - 8y^3$



expl 5: Factor completely.

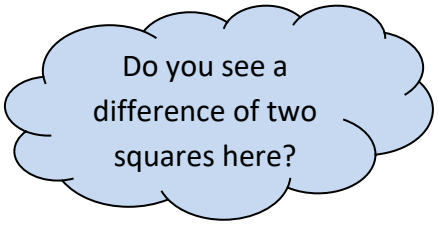
$b^3 + 64$



Many problems will need to be factored twice. Always look for additional factoring possibilities when you think you are done.

expl 6: Factor completely.

$16x^4 - 81$

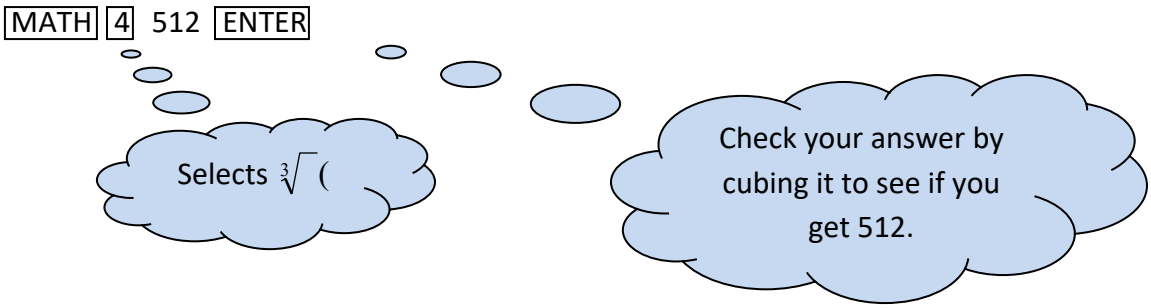


Calculator note: Square roots, cube roots: You will often want to know the cube or square root of a number.

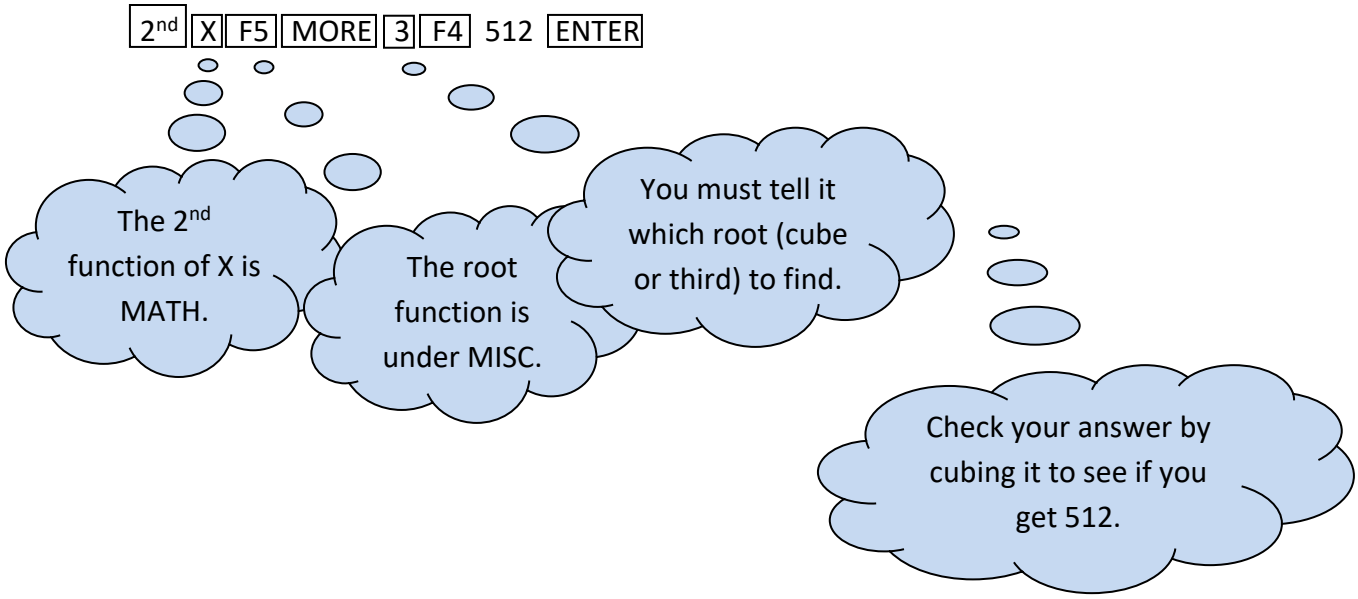
On the TI-82, 83, and 84, press the MATH button and you will find two useful functions, $\sqrt[3]{}($ and $\sqrt[x]{}$. The first is the cube root function. The second will be used to find other roots, such as the fourth or tenth roots.

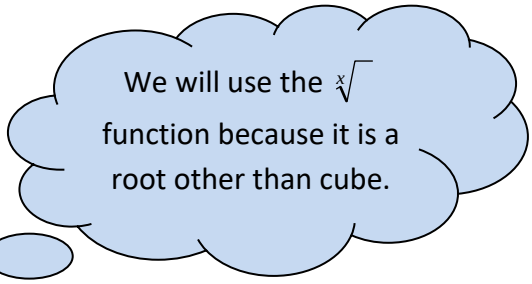
On the TI-86, enter the MATH menu by pressing the 2nd button and then the multiplication button. You will find the $\sqrt[x]{}$ under the MISC menu (once you are in the MATH menu, press F5 and then MORE.) This will find all roots, such as the cube or tenth root. There is no specific cube root function like the calculators above have.

expl 7 (TI-82, 83, and 84): Find the cube root of 512. Enter the following.



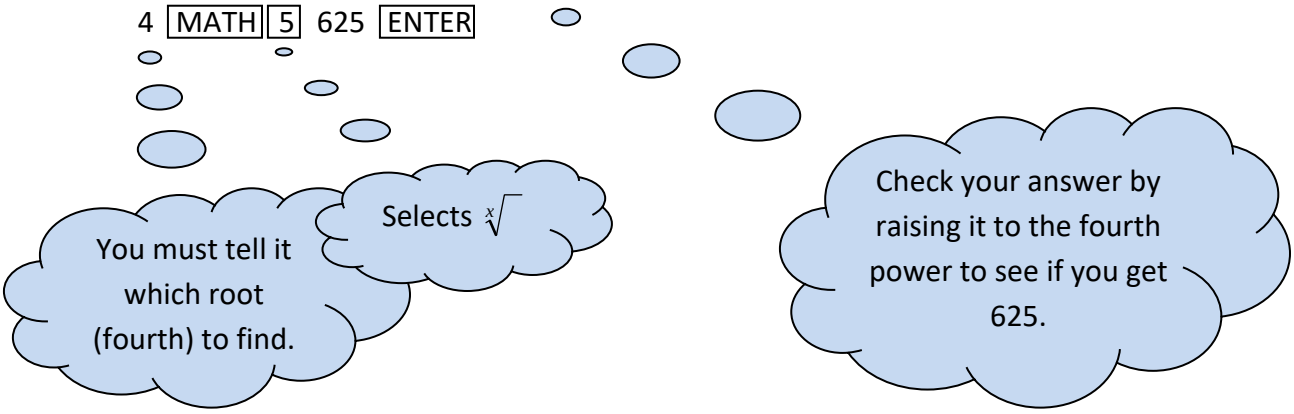
expl 7 (TI-86): Find the cube root of 512. Enter the following.





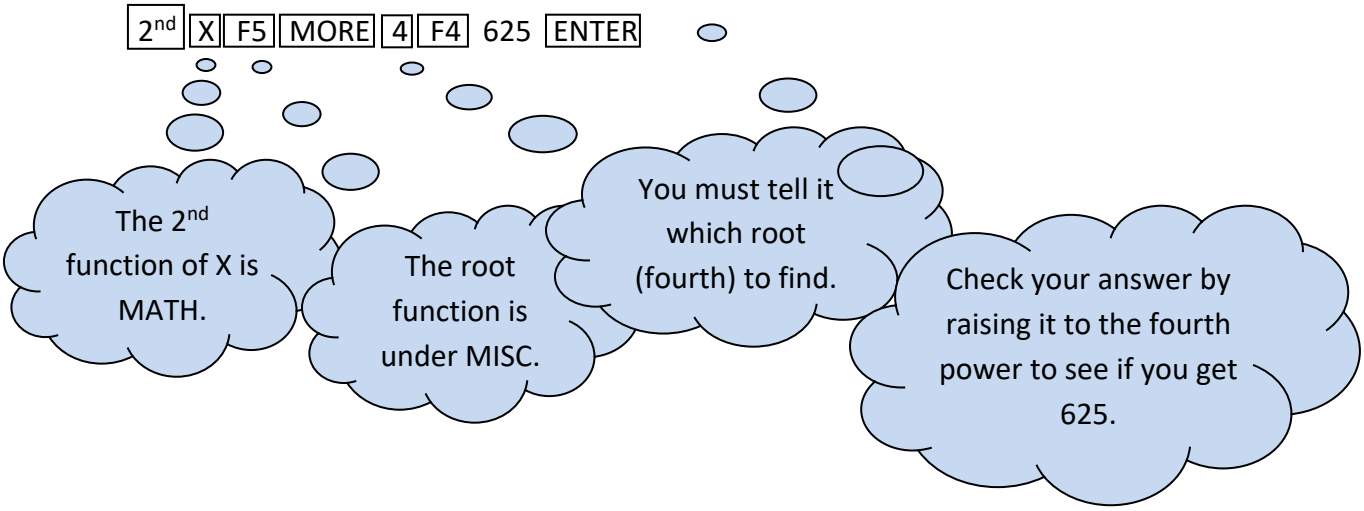
expl 8 (TI-82, 83, and 84): Find the fourth root of 625. Enter the following.

4 [MATH] [5] 625 [ENTER]



expl 8 (TI-86): Find the fourth root of 625. Enter the following. (This assumes you have exited the menu after the last example. Notice you can start at the 4 if you have not.)

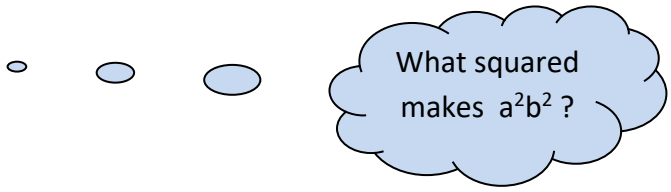
[2^{nd}] [X] [F5] [MORE] [4] [F4] 625 [ENTER]



Calculator note: Raising numbers to exponents: Remember to raise numbers to a power, we use the [^] button or the [x^2] (to square numbers) button.

expl 9: Factor completely.

$a^2b^2 - 49$



expl 10: Factor completely.

$4x^6 - 9y^2$