> We will use the formula to solve equations and glean other valuable information from equations.

Intermediate algebra
Class notes
Solving Quadratic Equations by the Quadratic Formula (section 18.2)

Recall: Definition: Quadratic equation: A quadratic equation is an equation that could be written in the form $ax^2 + bx + c = 0$ where a is *not* zero.

To solve a quadratic equation (like any equation), we need to isolate the x. We have done this in the past by factoring, using the square root property, and completing the square.

Some wonderful soul (many souls, really, over the years) actually solved the generic equation $ax^2 + bx + c = 0$ for x. They ended up with what we call the quadratic formula.

> How would you isolate x?

Definition: Quadratic Formula: A quadratic equation in the form $ax^2 + bx + c = 0$ (where a is *not* zero) has the solutions $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

expl 1: Solve.

$$x^2 + 10x + 13 = 0.$$

> What are a, b, and c? ($ax^2 + bx + c = 0$)

> Always start by writing the quadratic formula. Then substitute your values for a, b, and c.

> Mind your order of operations!

> Leave your answer in reduced radical form.

What are a, b, and c?

expl 2: Solve.

$m^2 + m + 7 = 0$

What happens under the radical? Are the solutions real?

expl 3: Solve.

$x^2 - 6x - 7 = 0$

What are a, b, and c?
($ax^2 + bx + c = 0$)
Be careful!

Reduce your answers as far as you can.

expl 4: Solve.

$x^2 + 8x = 12$

What do you need to do before using the formula?

expl 5: Solve.

$(2x + 1)(x - 3) = -9$

What do you need to do before using the formula?

What happens under the radical?

Multiply by the LCD to make the formula easier.

expl 6: Solve.

$$\frac{1}{3}y^2 - y - \frac{1}{6} = 0$$

Finding *x*-intercepts:

Do you remember how to find the x-intercept of any function, no matter what its equation is? How do we find the x-intercept of the quadratic function $y = ax^2 + bx + c$?

Do you see an x-intercept here?

expl 7: My building is 50 feet high. I chucked a ball upward from the roof. It soared up and then down, down, down to the ground below. Its height h is given by $h = -16t^2 + 20t + 50$ where t is the number of seconds after chucking. How long will it take for the ball to hit the ground? Round to the nearest tenth of a second.

Definition: Discriminant: In the quadratic formula, the expression under the radical, $b^2 - 4ac$, is called the discriminant. It can be used to glean information about an equation's solutions without fully solving it.

Connection between the discriminant and the solutions to $ax^2 + bx + c = 0$:

The question is, "How many real solutions does an equation have if the discriminant is positive?" This happened, you'll recall, in examples 1, 3, 4, and 6 above.

What about when the discriminant is negative? How many real solutions did the equations in examples 2 and 5 have? What was the nature of the solutions?

What about when the discriminant is zero? What happens to the solutions $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ when $b^2 - 4ac$ is 0?

expl 8: Use the discriminant to determine the number and type of solutions to the equation below. In other words, how many solutions will there be and are they complex (but not real) or just real numbers?

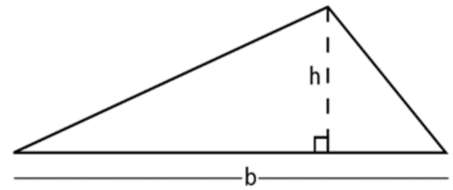
$3 - 6x^2 = -5x + 1$

What are a, b, and c?
($ax^2 + bx + c = 0$)
Be careful!

Only calculate $b^2 - 4ac$. Do not use the whole formula.

Since the solution(s) to $0 = ax^2 + bx + c$ are the x-intercepts of the function $y = ax^2 + bx + c$, this information also tells us about the graph of $y = ax^2 + bx + c$.

expl 9: The area A of a triangle is given by $A = \frac{1}{2}bh$ where b is the base length and h is the height of the triangle (shown at right). If the base of my triangle is two less inches than three times the height and the area of the triangle is 32.5 square inches, find the lengths of the base and height.



We will spend some time programming our calculators for the experience. Below are the handouts that will be given in class.

Calculator Program: QUADFORM
This program will calculate the real solutions to a quadratic equation. You will learn how to program your calculator and use calculator programs which are valuable skills. After you practice the quadratic formula by hand several times, feel free to use the calculator to solve equations. Be aware that the answers will appear as approximated decimals. Often, the book requires the exact radical form. Some calculation will be required to check your answers against the book.

Worksheet: Instructions for Programming Your Calculator (TI82, 83, 84, 85, 86)
This has general instructions on how to enter the program screen and where to find the various commands the program calls for.

Worksheet: Quadratic Program for the TI83
This is the program itself. It will work for the TI84 as well as the TI83. You will need to enter the program exactly as it appears on paper. If you have another TI calculator, you will need to get the program from me; it is not available on the Website. If you have a Casio or a Sharp calculator, I also have a program that may work for you. Come see me.

Worksheet: QUADFORM Program Instructions
This document shows you how to use the program once you put it in your calculator. It works through solving an example quadratic equation.

We are not programming our calculator to solve for complex solutions. That program is available but too complicated for us to program ourselves at this time. However, the program above will calculate the discriminant and tell you if the solutions are complex (non-real). You can then complete the formula to find the exact solutions if necessary.