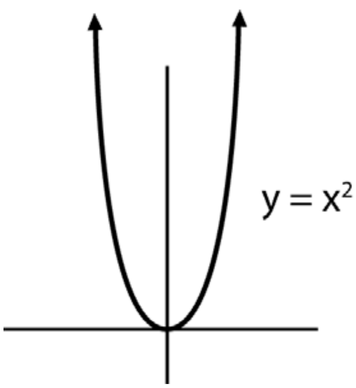
We will investigate the shape, vertex, orientation, and axes of symmetry of these functions.

Intermediate algebra
Class notes
Graphs of Quadratic Functions (section 18.5)

**Definition: Quadratic Function:** A quadratic function is a function that could be written in the form $f(x) = ax^2 + bx + c$ where $a$ is *not* zero. We will also see the form $f(x) = a(x - h)^2 + k$ where $a$ is *not* zero. (Completing the square will get us from the first form to the other.)

We will investigate the graphs of quadratic functions such as $y = (x+5)^2$, $f(x) = 4x^2$, and $g(x) = -3(x-4)^2 + 9$. These can be considered variations on the basic quadratic function $y = x^2$. So let's start there…

Here is the basic quadratic function $y = x^2$. Notice the nice symmetric shape. Also, notice how its "bend" is exactly on the origin.



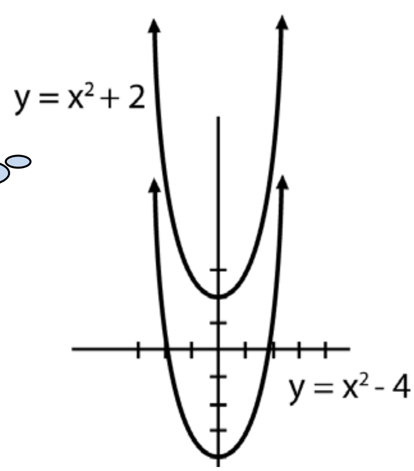

This shape is called a **parabola**. Its **vertex** is where (in ordered pair form) it bends. Label it in ordered pair form.

**Definition: Vertex and Axis of Symmetry:** The **vertex** of a parabola is where it bends. We will use ordered pair notation to write the vertex. The **axis of symmetry** is the vertical line that goes through the vertex; it would be in the form $x$ = some number. Notice the graph is symmetric about that imaginary line.

**Variation 1: Graphs of $y = x^2 + k$ and $y = x^2 - k$:**

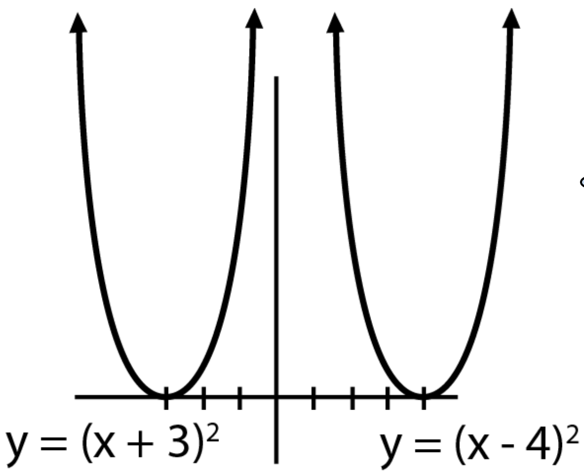
Consider the graphs here of $y = x^2 + 2$ and $y = x^2 - 4$. How do those graphs compare to the graph of $y = x^2$?

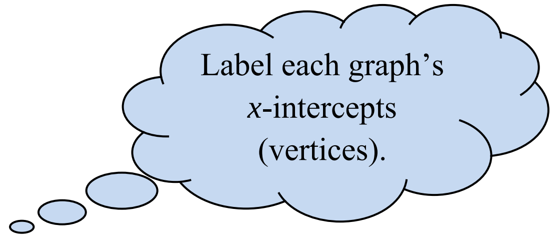
Label each graph's *y*-intercepts (vertices).



**Variation 2: Graphs of $y = (x + h)^2$ and $y = (x - h)^2$:**

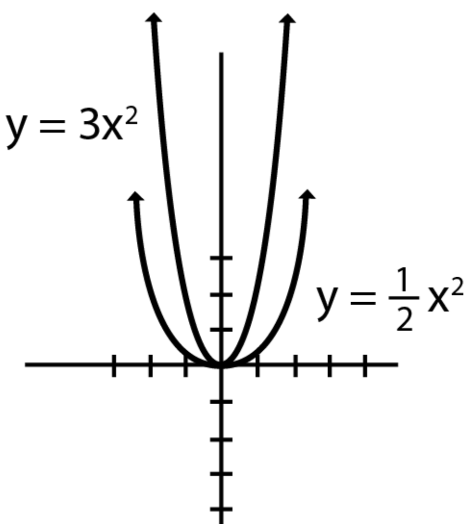
Consider the graphs here of $y = (x + 3)^2$ and $y = (x - 4)^2$. How do those graphs compare to the graph of $y = x^2$?

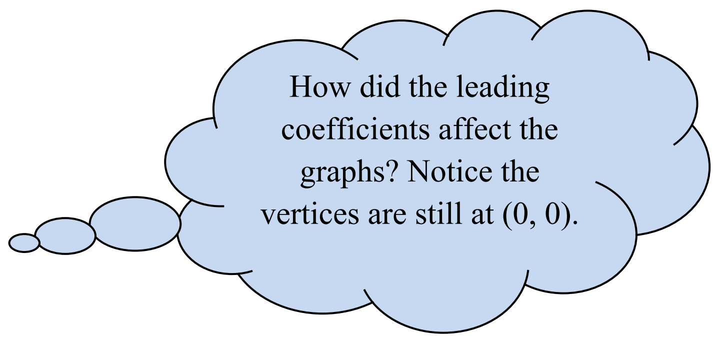



**Variation 3: Graphs of $y = ax^2$ where $a$ is positive:**

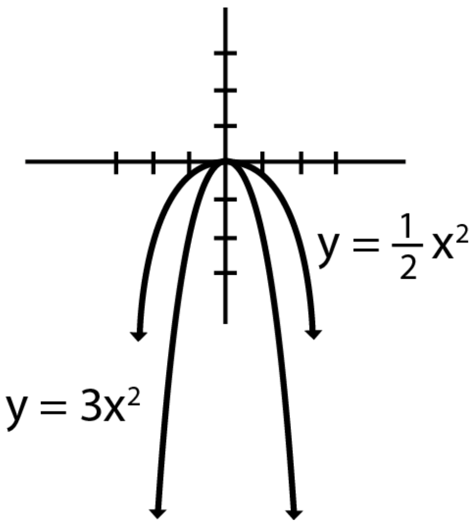
Consider the graphs here of $y = 3x^2$ and $y = \frac{1}{2}x^2$. How do those graphs compare to the graph of $y = x^2$?



**Variation 4: Graphs of $y = ax^2$ where $a$ is negative:**

Consider the graphs here of $y = -3x^2$ and $y = -\frac{1}{2}x^2$. How do those graphs compare to the graph of $y = x^2$?




How did the negative leading coefficients affect the graphs? Again, the vertices are at (0, 0).

**Summary of Variations on the Graph of $y = x^2$:**

**Variations 1 and 2: Vertical or Horizontal Shifts:** So, adding or subtracting a number to $y = x^2$ moves the whole graph up or down (variation 1). If we add or subtract a number **before** we square, the graph is moved left or right (variation 2).

**Variation 3: Vertical Compressions or Stretches:** If the leading coefficient is greater than 1, the graph is narrower than that of $y = x^2$. The graph is stretched. If the leading coefficient is between 0 and 1, the graph is wider than that of $y = x^2$. We say that it is compressed (or squashed).

**Variation 4: Reflection over the $y$-axis:** If the leading coefficient is negative, the graph is flipped over the $y$-axis. If it is less than -1, the graph will be narrower than that of $y = x^2$ (because it is also stretched). If it is between 0 and -1, the graph will be wider than that of $y = x^2$ (because it is also compressed).

You may see these rules stated a bit differently. Let's use these facts to graph some quadratic functions by hand.

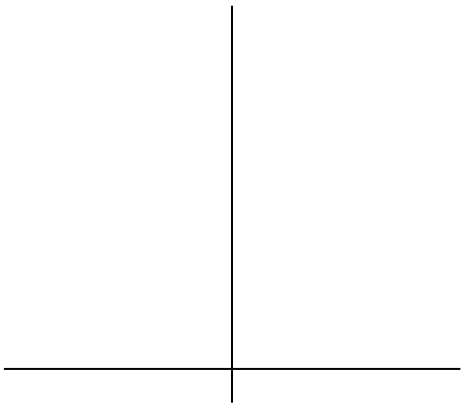
Use what you know to plot the vertex. Then, substitute an $x$-value close to the vertex to plot a second point. Connect them with a smooth curve. Mimic that shape to complete the other half of the parabola. You will also need to sketch and label the axis of symmetry. That is merely the vertical line through the vertex.

expl 1: Sketch the graph of the function below. Label the vertex. Sketch (as a dashed line) and label the axis of symmetry.

$f(x) = x^2 + 7$

Variation 1

The vertex of $y = x^2$ is (0, 0). Move it up 7 units to start drawing $f(x) = x^2 + 7$.

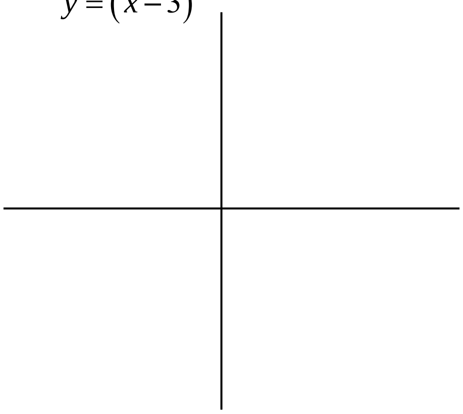


expl 2: Sketch the graph of the function below. Label the vertex. Sketch (as a dashed line) and label the axis of symmetry.

$y = (x-3)^2$

Variation 2

The vertex of $y = x^2$ is (0, 0). Move it to the right 3 units to start drawing $y = (x-3)^2$.



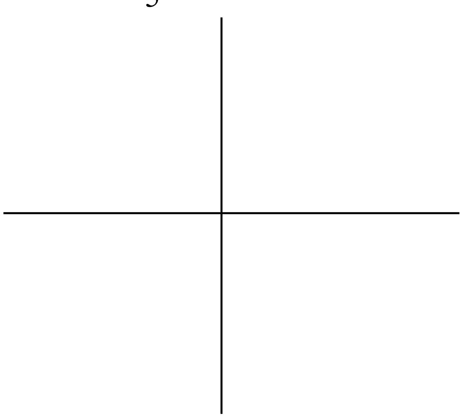
expl 3: Sketch the graph of the function below. Label the vertex. Sketch (as a dashed line) and label the axis of symmetry.

$g(x) = \frac{1}{5}x^2$

Variation 3

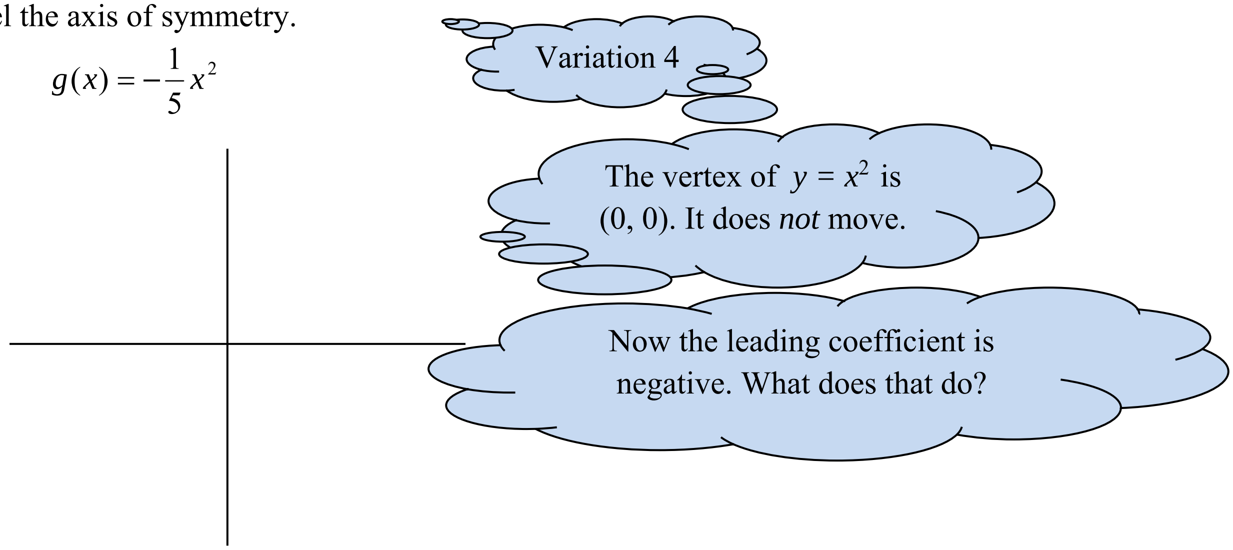
The vertex of $y = x^2$ is (0, 0). It does *not* move.

How does the $\frac{1}{5}$ affect the graph?



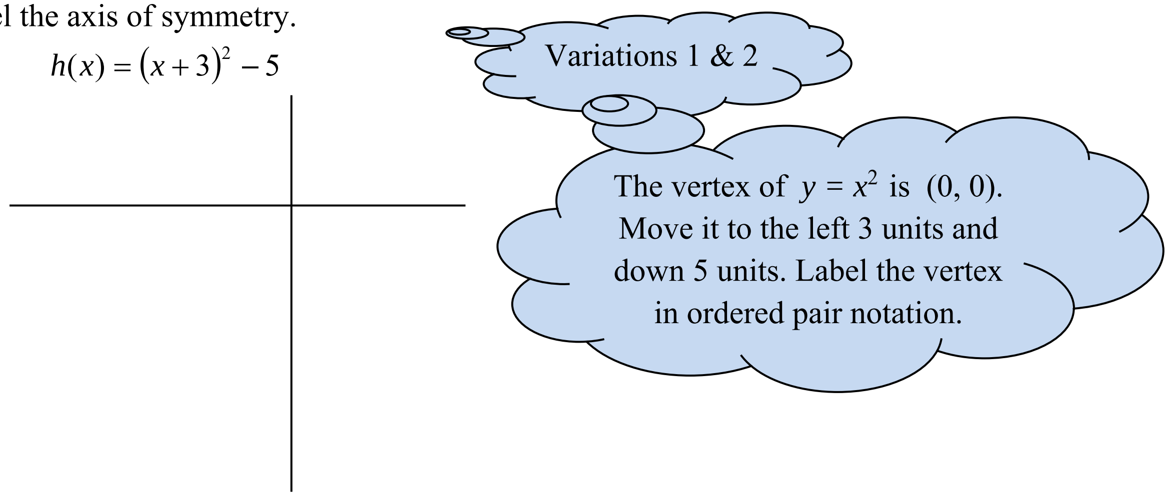
expl 4: Sketch the graph of the function below. Label the vertex. Sketch (as a dashed line) and label the axis of symmetry.

$$g(x) = -\frac{1}{5}x^2$$



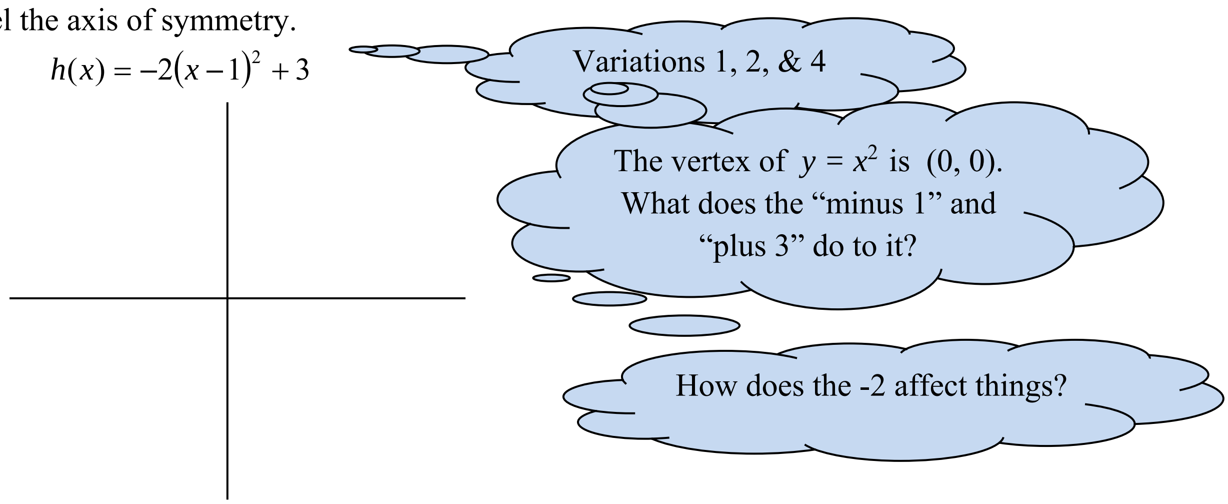

expl 5: Sketch the graph of the function below. Label the vertex. Sketch (as a dashed line) and label the axis of symmetry.

$$h(x) = (x+3)^2 - 5$$




expl 6: Sketch the graph of the function below. Label the vertex. Sketch (as a dashed line) and label the axis of symmetry.

$$h(x) = -2(x-1)^2 + 3$$



**Combination of variations:** Think back to example 6. It is in the form $f(x) = a(x-h)^2 + k$ . Its vertex is ($h$, $k$). The axis of symmetry is $x = h$.

If $a < 0$, the parabola opens downward.
If $a > 0$, the parabola opens upward.

Does your graph in example 6 agree?

Remember to get an accurate graph, not only do you need to use the facts we discussed here, but you will also need to plot at least one point near the vertex. This gives you an idea of the parabola's exact shape. Simply substitute an $x$-value into the formula to calculate the corresponding $y$-value, and plot that point.

Do *not* forget to sketch and label the axis of symmetry.

expl 7: Let's try one in MML to get an idea of how the grapher will work.