We will work with the vertex, orientation, and x- and y-intercepts of these functions.

Intermediate algebra
Class notes
More Graphs of Quadratic Functions (section 18.6)

In the previous section, we investigated the graphs of these functions by thinking of them as variations of the basic $y = x^2$ graph. Here, we will take a different approach. We will be given a formula that will tell us the graph's vertex (in ordered pair notation). We will then find the graph's x- and y-intercepts algebraically. This information along with the orientation of the parabola will allow us to graph it by hand.

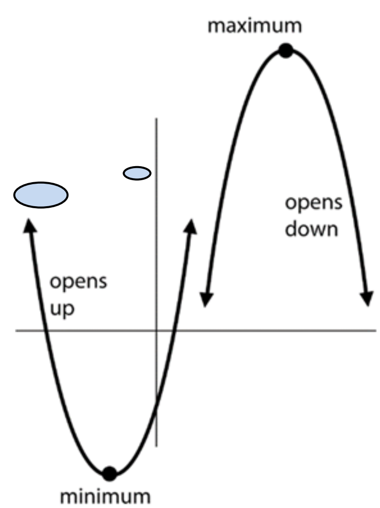
Vertex of a Parabola: If a quadratic function is in the form $f(x) = ax^2 + bx + c$, the x-value of the vertex can be found by calculating $x = \frac{-b}{2a}$.

But a vertex has **two coordinates**, an x and a y coordinate.
So, how would you find the corresponding y-value?

This is usually written as
$vertex = \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

Recall: Orientation of a Parabola:
If $a < 0$, the parabola opens downward.
If $a > 0$, the parabola opens upward.

Orientation describes whether the parabola opens up or down.



Alternative Formula for Vertex of a Parabola: If a quadratic function is in the form $f(x) = a(x-h)^2 + k$, its vertex will be (h, k).

Graphing by hand: We will graph quadratic functions by calculating and plotting the vertex, and x- and y-intercepts. We will then use the orientation (and its symmetry) to fill in the rest of the parabola.

Recall: Finding *x*- and *y*-intercepts: To find the x-intercept of a function, substitute 0 for y and solve for x. To find the y-intercept, substitute 0 for x and solve for y. (This is true for any type of function, *not* just quadratic ones.) Finding the x-intercept may require the quadratic formula. That is a good use of the calculator program since it gives decimal answers.

Finding the vertex: We have two different forms of a quadratic function, $y = a(x-h)^2 + k$ and $y = ax^2 + bx + c$. You may see examples where you are given the second form, but are told to change it to the first form (by completing the square) so that you can use the fact that the vertex is (h, k). You do *not* need to go this route but can instead use the fact that the vertex for $f(x) = ax^2 + bx + c$ is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$. Let's do some examples.

expl 1: Find the vertex of the quadratic function.

$y = -x^2 + 3x - 4$

This is in the form $y = ax^2 + bx + c$. So calculate the x-value of the vertex as $x = \frac{-b}{2a}$. How would you find the y-value that goes with it?

Write the vertex in ordered pair notation.

expl 2: Find the vertex of the quadratic function.

$g(x) = 3(x+4)^2 - 5$

This is in the form $f(x) = a(x-h)^2 + k$. So pick the vertex out as (h, k). What is h? What is k?

Graphing Quadratic Functions by Hand: Plot the vertex and the x- and y-intercepts. Then use what you know about the parabola's orientation to complete the sketch.

How do know a
parabola's orientation?

expl 3: You have found this function's vertex. Now find its orientation and x- and y-intercepts to sketch its graph.

$$y = -x^2 + 3x - 4$$

Use QUADFORM to
solve $0 = -x^2 + 3x - 4$.

What does "no real
solutions" mean?

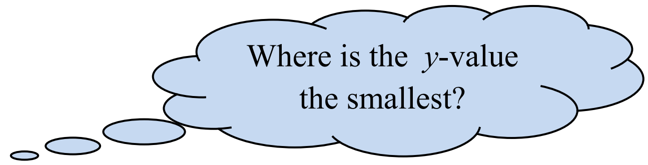
expl 4: You have found this function's vertex. Now find its orientation and x- and y-intercepts to sketch its graph.

$$g(x) = 3(x + 4)^2 - 5$$

We'll find $g(0)$ and also
solve $0 = 3(x + 4)^2 - 5$.
Do you see why?

Minimum and Maximum Values: Let's look at these graphs in more detail. Use your calculator to check your graph of $g(x) = 3(x+4)^2 - 5$. Use the Standard Window.

Use the TRACE button to trace along the graph, watching the y-values decrease and then increase as you go left to right. Try to home in on the smallest y-value. This gives you an idea of what the smallest y-value is but it is *not* as accurate as we would like.



Worksheet: Finding Maximums and Minimums on Your Calculator (82, 83, 85, 86):
This worksheet will show you how to get the calculator to calculate the **exact** vertex of quadratic functions. (It also works to find maximums and minimums of other types of functions seen later in math.) This will be more accurate than what the TRACE button will get you. The instructions will also work for the TI84. You can ignore #6 on the worksheet.

expl 5: Find the exact vertex for $g(x) = 3(x+4)^2 - 5$ using the calculator. Does it match what you calculated in example 4?

Optional Worksheet: Quadratic functions: Maximums and Minimums Practice:
We will practice the vertex formula given in this section and also investigate an application that uses linear functions to help define a quadratic function whose maximum we will then find. Solutions are available online.

Optional Worksheet: Quadratic functions practice:
We will practice finding the orientation, y-intercepts, vertices of quadratic functions as well as sketching their graphs. You will also touch on solving quadratic equations graphically. Solutions are available online.

There are some pretty interesting applications for this.

expl 6: A ball is thrown upward off the top of a building. Its height in feet $h(t)$ is given by the equation $h(t) = -16t^2 + 35t + 200$ where t is the number of seconds after launch. Find the maximum height the ball achieves. How many seconds does it take to achieve this maximum? Include a quick graph below with the vertex labeled.

Which part of the graph corresponds to the path of the ball?

The vertex will be a maximum. How do you know that?

Graph on your calculator. Use x as the input variable, *not* t. Increase the y-max until you see the vertex.

expl 7: A length and width of a certain rectangle has a sum of 25. Find the dimensions of the rectangle that will have the maximum area. Include a quick graph below with the vertex labeled.

We know $l + w = 25$; solve that for l. Now, area is $A = l \cdot w$. Substitute your expression to get area in terms of only its width. Graph that sucker!