Think about like terms as you add or subtract radicals. The multiplication problems will need the distribution property or FOIL.

Intermediate algebra
Class notes
Adding, Subtracting, and Multiplying Radical Expressions (section 17.4)

The most important thing to keep in mind is something with which you are already familiar. Recall how you would simplify $6x + 3y + 2x$ or $(4x+2)(x-3)$. Do them now.

In this section, we will do mostly the same stuff when we see radical expressions like $6 \cdot \sqrt[3]{4} + 3 \cdot \sqrt[3]{6} + 2 \cdot \sqrt[3]{4}$ or $(4\sqrt{10}+2)(\sqrt{10}-3)$. Sometimes we will need to simplify individual terms by reducing the radicals within, but then we will combine like terms as we have done before.

Definition: Like Radicals: Like radicals are radicals with the same index and the same radicand. We treat them similarly to like terms.

As an example, in the expression $6 \cdot \sqrt[3]{4} + 3 \cdot \sqrt[3]{6} + 2 \cdot \sqrt[3]{4}$, the first and third terms are like radicals. We can add them as we would add $6x + 2x$.

Recall, the distribution property is the reason combining like terms works. We see this with radicals too.

$$
\begin{aligned}
&6 \cdot \sqrt[3]{4} + 2 \cdot \sqrt[3]{4} \\
&= (6+2) \cdot \sqrt[3]{4} \\
&= 8 \cdot \sqrt[3]{4}
\end{aligned}
$$

$6x + 2x$
$= (6 + 2)x$
$= 8x$

Just like how the *y*-term remains unchanged in $6x + 3y + 2x$, so does the non-like radical in $6 \cdot \sqrt[3]{4} + 3 \cdot \sqrt[3]{6} + 2 \cdot \sqrt[3]{4}$. After combining the first and third terms, the expression becomes $8 \cdot \sqrt[3]{4} + 3 \cdot \sqrt[3]{6}$.

Many examples will contain radicals that need to be simplified first, before combining like radicals.

Look for perfect squares, cubes, *n*th powers.

expl 1: Add or subtract.

$$9 \cdot \sqrt[3]{4} - 15 \cdot \sqrt[3]{4}$$

expl 2: Add or subtract.

$$7\sqrt{10} + 6\sqrt{15} - 12\sqrt{15} + 9\sqrt{10}$$

expl 3: Add or subtract.

$$6\sqrt{8} - 13\sqrt{18} + 9\sqrt{200}$$

All of these radicals can be simplified.

Look for perfect squares.

$$6\sqrt{8} = 6\sqrt{4 \cdot 2} = 6 \cdot 2\sqrt{2} = 12\sqrt{2}$$

expl 4: Add or subtract.

$$2 \cdot \sqrt[3]{3a^4} - 3a \cdot \sqrt[3]{81a}$$

Look for factors of $3a^4$ and $81a$ that are perfect cubes.

Multiplying radicals: When we multiply radicals, we will also need to combine like terms just as you do with FOIL problems like $(4x+2)(x-3)$. You will find we also use the $\sqrt[n]{a}\cdot\sqrt[n]{b}=\sqrt[n]{a\cdot b}$ rule a lot.

expl 5: Multiply and simplify if possible.

$$\sqrt{98}\left(\sqrt{18}+\sqrt{10}\right)$$

$\sqrt{98\cdot 18}$

$=\sqrt{49\cdot 2\cdot 9\cdot 2}$

$=??$

expl 6: Multiply and simplify if possible.

$$\left(\sqrt{3}+\sqrt{8}\right)\left(\sqrt{6}-\sqrt{2}\right)$$

FOIL

expl 7: Multiply and simplify if possible. Assume variables are non-negative numbers.

$$\left(\sqrt{x}+y\right)\left(\sqrt{x}-y\right)$$

Do you recognize this form?

expl 8: Multiply and simplify if possible.

$$\left(\sqrt[3]{x}+1\right)\left(\sqrt[3]{x}+2\right)$$

expl 9: Multiply and simplify if possible. Assume variables are non-negative numbers.

$$\left(\sqrt{2x}+4\right)^2$$

Write it out longhand first.