

Rationalizing means to eliminate the radical from the bottom or top of a fraction.

Intermediate algebra

Class notes

Rationalizing Denominators and Numerators of Radical Expressions (section 17.5)

Rationalizing a denominator or numerator involves multiplying by something so that the radical is eliminated. Of course, you cannot change the value of the fraction. So whatever we do to the bottom of the fraction, we will also do to the top.

expl 1: Rationalize the denominator.

$$\frac{3x}{\sqrt{5}}$$

What could you multiply  $\sqrt{5}$  by so that it is no longer a radical?

Remember, you cannot just multiply by something on bottom. You must do the same to the top.

expl 2: Rationalize the denominator.

$$\frac{\sqrt{2x}}{\sqrt{5y}}$$

Think of it as  $\frac{\sqrt{2x}}{\sqrt{5y}}$ .

What could you multiply  $\sqrt{5y}$  by so that it is no longer a radical?

Turn the  $5y$  into a perfect square.

expl 3: Rationalize the denominator.

$$\frac{5a}{\sqrt[5]{8a^9b^{11}}}$$

We need to turn the  $8a^9b^{11}$  into a perfect 5<sup>th</sup> power.

Let's consider a more complicated expression to rationalize. What would you need to multiply  $(\sqrt{5} + 3)$  by in order to get rid of the radical? Let's try a few things.

Does multiplying by  $\sqrt{5}$  eliminate the radical? What about  $\sqrt{3}$ ?

What about  $-\sqrt{5}$ ?

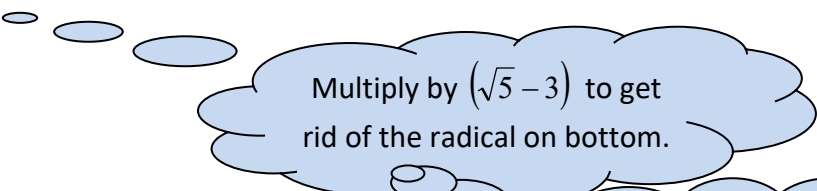
What about  $(\sqrt{5} + 3)$ ?

What about  $(\sqrt{5} - 3)$ ?

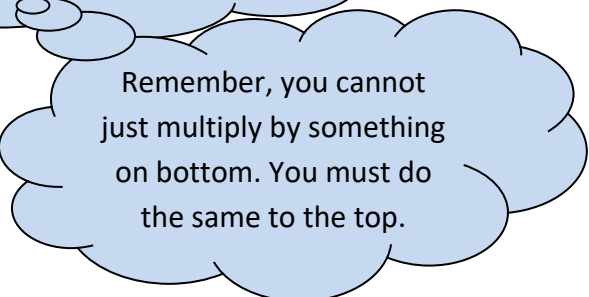
**Definition: Conjugate:** Two expressions are conjugates if one is the sum of two numbers and the other is the difference. The expressions  $a + b$  and  $a - b$  are conjugates. The expressions  $(\sqrt{5} + 3)$  and  $(\sqrt{5} - 3)$  are conjugates. They will be used to rationalize denominators and numerators.

expl 4: Rationalize the denominator.

$$\frac{2}{\sqrt{5} + 3}$$



Multiply by  $(\sqrt{5} - 3)$  to get rid of the radical on bottom.



Remember, you cannot just multiply by something on bottom. You must do the same to the top.

expl 5: Rationalize the denominator.

$$\frac{2\sqrt{a} - 3}{2\sqrt{a} + \sqrt{b}}$$

What is the conjugate  
of  $2\sqrt{a} + \sqrt{b}$  ?

Don't forget  
to FOIL!

**Rationalizing numerators:** We do the same thing when asked to rationalize a numerator, but we do it to the numerator.

expl 6: Rationalize the numerator.

$$\frac{\sqrt{3x^5}}{6}$$

How do you change  
 $3x^5$  into a perfect  
square?

It does not matter if  
there is a radical in  
the bottom when we  
are done.

expl 7: Rationalize the numerator.

$$\frac{\sqrt{x} + 3}{\sqrt{x}}$$

Don't forget  
to simplify in  
the end.