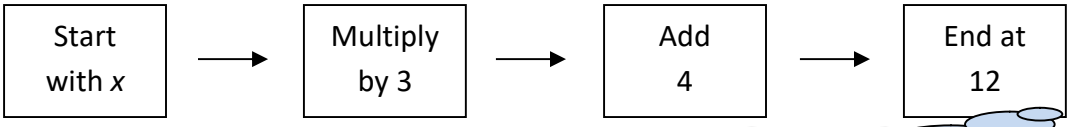
To solve an equation, undo what was done to the variable.

Intermediate algebra
Class notes
Solving Radical Equations and Problem Solving (section 17.6)

Main idea: To solve most linear equations (and some radical equations), you can think of undoing what was done to x to form the equation. Thinking about it this way takes a little practice but can be useful in determining the steps needed to isolate the variable. An example of this follows.

expl: Solve $3x + 4 = 12$

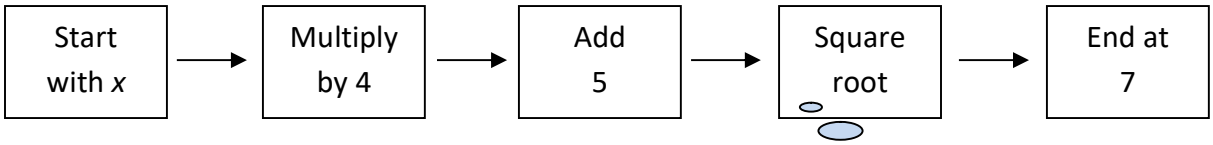
What happened to the x to make the equation in the first place?



Solving an equation involves undoing these things. We work backward through this verbal model.

Recall that you would subtract 4, and then divide by 3 to solve this equation. We can think of the subtracting as undoing the "add 4" and the division undoing the "multiply by 3" step.

Let's consider a radical equation we are asked to solve, $\sqrt{4x+5} = 7$. Its verbal model follows. To solve the equation, you would undo the steps to reveal x.



How do you undo the square root?

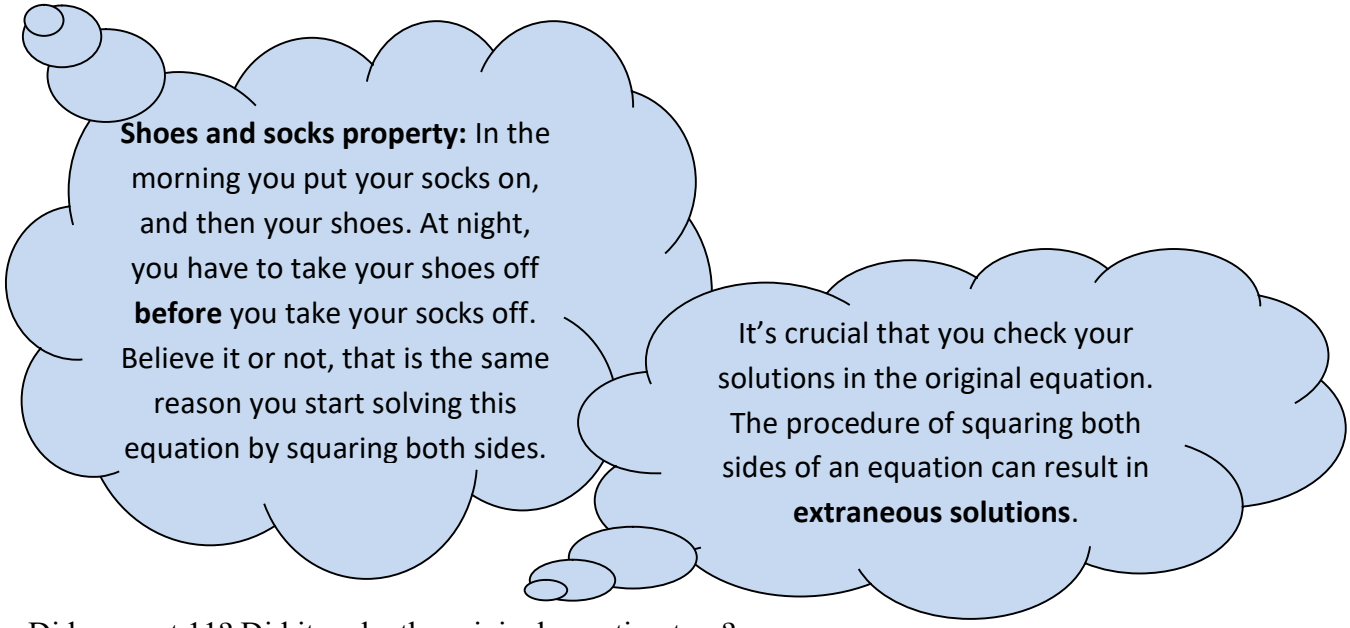
What undoes square rooting?

Subtraction undoes addition.

Division undoes multiplication.

Square root 16 and get 4. What do we do to 4 to get back to the 16?

Solve $\sqrt{4x+5}=7$ by undoing what was done to x. The key is to do the operations in the proper order. So pay attention to the verbal model. Check your answer in the **original** equation.



Did you get 11? Did it make the original equation true?

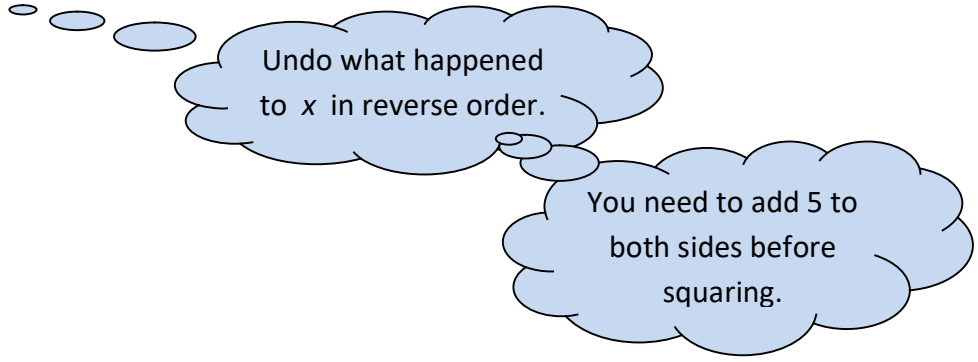
Let's try a few more equations that have one instance of the variable. Then we will take what we learned from them to attack more complicated equations.

expl 1: Solve.

$\sqrt{x-4}=4$

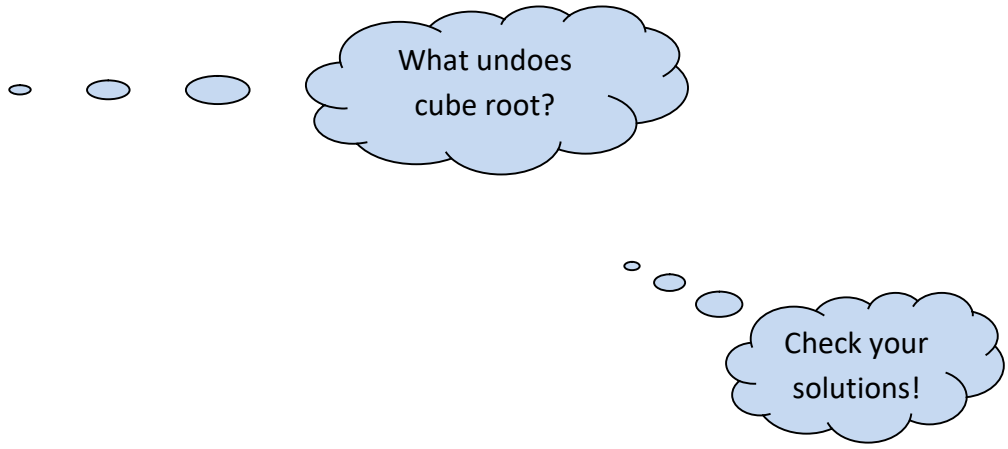
expl 2: Solve.

$\sqrt{4x-3}-5=0$



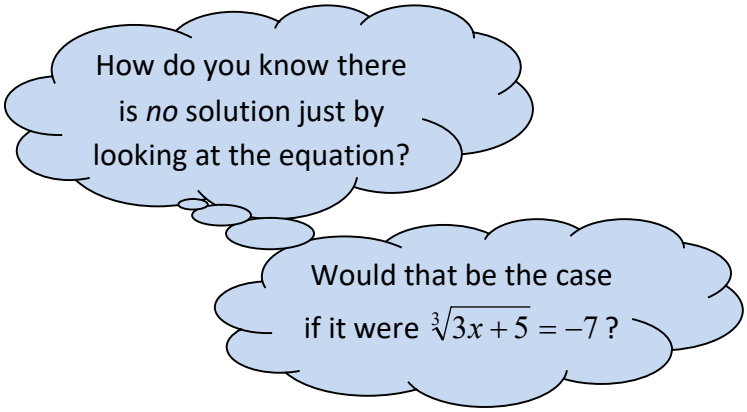
expl 3: Solve.

$\sqrt[3]{2x+15}=-1$



expl 4: Solve.

$\sqrt{3x+5} = -7$



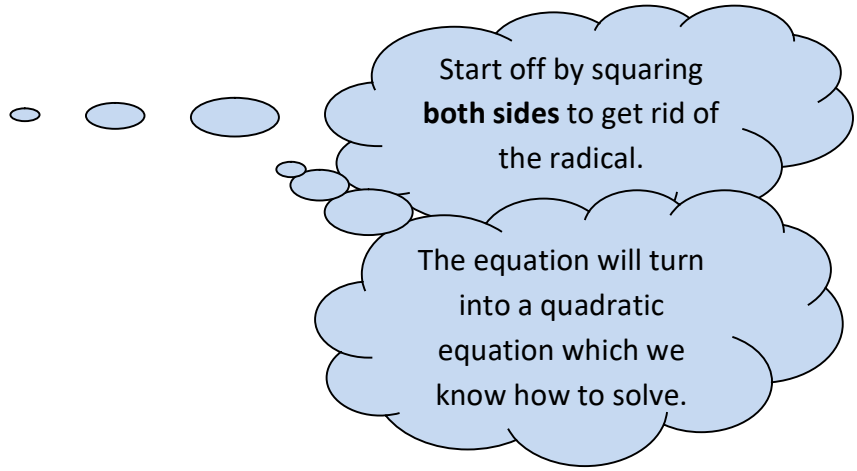
Worksheet: Solving radical equations with verbal models:
This worksheet will explore using verbal models to decipher the order of the steps needed to solve radical equations. We will also explore radical equations with no solutions.

So, from these examples, we learn that we can undo square roots by squaring, undo cube roots by cubing, and in general, undo nth roots by raising the radical to the nth power.

Let's solve more complicated equations. Since they have more than one instance of the variable, the verbal model process does *not* work so neatly. But we can use what we learned to unbury the x from underneath radicals.

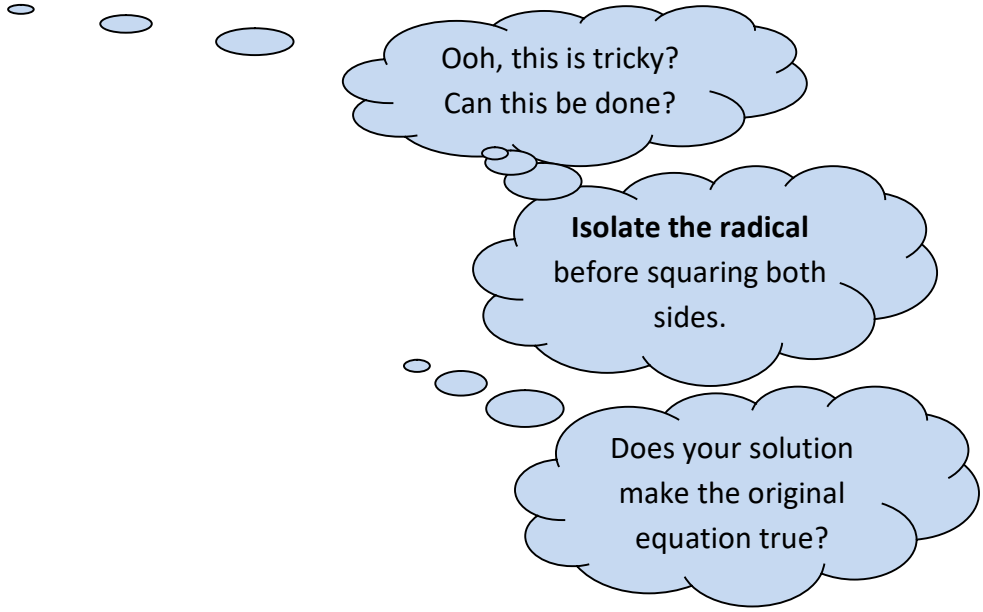
expl 5: Solve.

$\sqrt{13-x} = x-1$



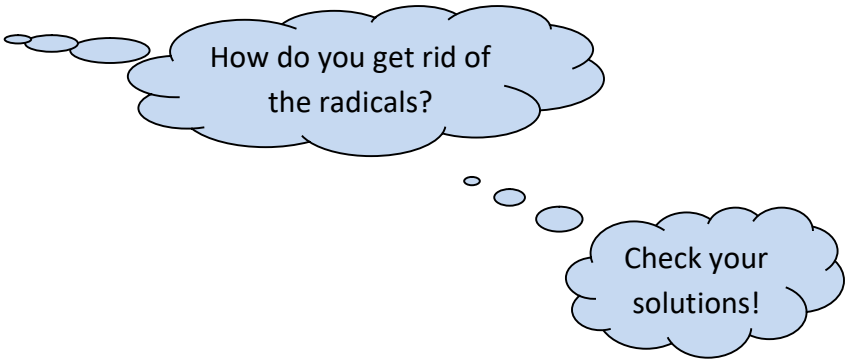
expl 6: Solve.

$$x - \sqrt{4 - 3x} = -8$$



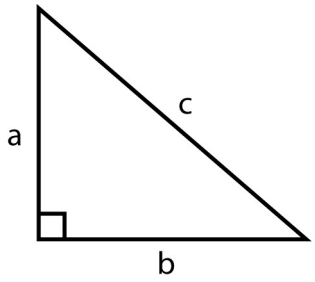
expl 7: Solve.

$$\sqrt[3]{-6x - 1} = \sqrt[3]{-2x - 5}$$

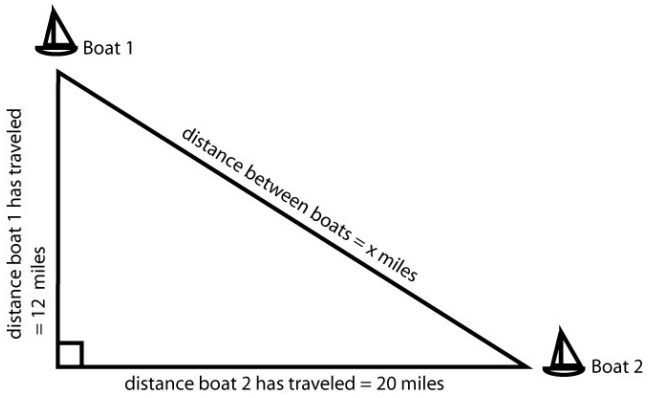


Story problems involving the Pythagorean Theorem:

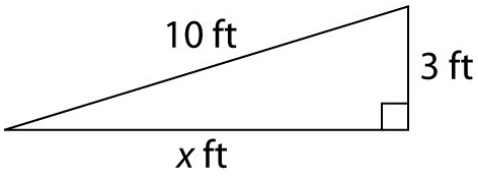
Write the Pythagorean Theorem using the right triangle here.



expl 8: One boat travels north 12 miles from the shore. Another boat travels east 20 miles from the shore. How far are they from each other? Refer to the picture.



expl 9: Find the missing leg of the right triangle pictured below.



Problems involving formulas: You will be given a formula. The most important thing is to keep straight what the variables represent. You will be given a value for one (or more) and asked to solve for another.

expl 10: The formula $v = \sqrt{2gh}$ gives the velocity v, in feet per second, of an object when it falls h feet accelerated by gravity g, in feet per second squared. Assume g is approximately 32 feet per second squared. Find how far an object has fallen if its velocity is 80 feet per second.