Intermediate algebra
Class notes
Rational Expressions and Functions (section 14.1)

How would you simplify a normal fraction like $10/25$? Whenever you get stuck, think back to how you deal with fractions with plain numbers.

Recall fractions:

Reduce $10/25$. Why does it work?

"simplify"
"reduce"
"write in lowest terms"

We see that $10/25$ could be thought of as $\frac{5\cdot 2}{5\cdot 5}$. But since there is a **common factor** of 5 on top and bottom, we can cancel them to get $2/5$.

$\frac{5}{5}$ is just 1

Does the same work for **common terms** on top and bottom? What about $\frac{7}{10}=\frac{5+2}{5+5}\stackrel{?}{=}\frac{2}{5}$? Can you cancel the common **term** here? Is $\frac{7}{10}$ equal to $\frac{2}{5}$?

So how would we simplify $\frac{x^2+5x+6}{x^2-x-12}$?

Can't cancel common terms but maybe we could factor both the top and bottom and then cancel. Do it now.

$\frac{x+3}{x+3}$ is just 1

Worksheet: Simplifying rational expressions: Cancelling common factors

Definition: Rational expression: A rational expression is a fraction where both the top and bottom are polynomials (exponents on x's are whole numbers).

expls: $\frac{x^2+5x+6}{x^2-x-12}$ $\quad \frac{4t^5-8t}{2}$ $\quad \frac{5x^2+3x-7}{3x^3+1x-3x^2-12}$ $\quad \frac{d^2+4d+4}{2d+3}$

counterexpls: $\frac{x^2+\sqrt{5x}+6}{x^{-2}-x-12}$ $\quad \frac{4t^{1/5}-8t}{2}$ $\quad \frac{\sqrt{5x^2+3x-7}}{3x^3+1x-3x^{-2}-12}$

Why are these *not* rational expressions?

Definition: Rational function: A rational function is a relationship between x and y where the right side is a rational expression. (Sometimes other variables are used.)

expls: $y=\dfrac{x^2+5x+6}{x^2-x-12}$ $\quad y=\dfrac{4t^5-8t}{2}$ $\quad y=\dfrac{5x^2+3x-7}{3x^3+1x-3x^2-12}$ $\quad y=\dfrac{d^2+4d+4}{2d+3}$

Functions often substitute f(x) or r(x) for the y.

$f(x)=\dfrac{x^2+5x+6}{x^2-x-12}$

$r(t)=\dfrac{4t^5-8t}{2}$

Definition: Domain of a rational function: The domain of any function is the set of x values that "work" in the function. That means these x values, when inputted, give you a valid y value out.

The only thing that stands in the way of an x value working is if it makes the bottom zero. So the domain of a rational function is said to be "all real numbers except those that make the bottom zero."

What is it called when the bottom of a fraction is 0?

In our work with rational expressions, we will simplify them, evaluate them (substitute values in for the variables), find their domain, and add/subtract/multiply/divide them (in later sections).

expl 1: Find the domain of $f(x)=\dfrac{x^2+5x+6}{x^2-x-12}$.

Solve $x^2-x-12=0$ to see where the bottom is 0.

expl 2: Evaluate $\dfrac{x+1}{x+13}$ for the value x = -5.

Put -5 in for x and simplify. Be sure to reduce if needed.

If it's undefined, say so.

expl 3: Find all values for which the following rational expression is undefined.

$$\frac{14}{x+7}$$

What is the domain of this expression? How is that different than what was asked?

expl 4: Find all values for which the following rational expression is undefined.

$$\frac{x+1}{x^2+11x+24}$$

Why does this matter?

expl 5: Simplify. (Assume the denominator is nonzero.)

$$\frac{x^2+6x-40}{x^2-10x+24}$$

expl 6: Check your "simplification" from example 5 by plugging a number into both the original and your "simplification". (Do *not* pick 0 or 1.) Do they yield the same result?

More cancelling of factors:

What is the relationship between the following pairs?

$5-7$ and $7-5$
$14-2\cdot3$ and $2\cdot3-14$
$x-3$ and $3-x$

What is true of all 3 pairs of numbers?

So it turns out that if you have a difference and you switch the order of subtraction, you get its opposite. Make up your own example with a variable. Then substitute any value in to see if they are truly opposites.

$2x-5$ and $5-2x$

Put 4 in for x to see if it works!

In the previous examples, if we had something like $\frac{x+3}{x+3}$ inside an expression, we could cancel them to get 1. If we have something like $\frac{x-3}{3-x}$ in an expression, we can cancel them to make -1. Let's see how this works.

expl 7: Simplify $\frac{(8+x)(8-x)}{x^2-2x-48}$.

Factor the bottom. Look for common or opposite factors.

$\frac{8-x}{x-8}$ simplifies to -1

Your answer could be written in many equivalent ways. Among them are $-\frac{8+x}{x+6}$ and $\frac{-8-x}{x+6}$ and $\frac{8+x}{-1(x+6)}$ and $\frac{-1(x+8)}{x+6}$. I prefer the last because the factor of -1 is more obvious.

expl 8: Evaluate r(8) for the function $r(t) = \frac{4t^2 - 8t}{2t + 3}$.

Put 8 in for t and simplify.

Try putting it all into your calculator at once. But be sure to put the whole top in parentheses and the whole bottom in parentheses (because they have multiple terms). Enter the following into your calculator.

$(4 * 8^2 - 8 * 8) / (2 * 8 + 3)$ ENTER

Just the multiplication key

Use the x^2 key on left.

Parentheses around top
Parentheses around bottom

Convert your final answer to a fraction (under the MATH menu) to match the book.

expl 9: Find the domain of the function $d(x) = \frac{5x^2 + 7x + 4}{x^2 - 64}$.

Solve $x^2 - 64 = 0$ to see where the bottom is 0. The domain is "all real numbers except" those.

Domain could be expressed in set builder notation:
{x | x is a real number and x ≠ ?, x ≠ ? }

expl 10: The total revenue from the sale of a book is approximated by $R(x) = \frac{750x^2}{x^2 + 7}$ where x is the number of years since publication and R(x) is the total revenue in millions of dollars. Find the total revenue of this book after 5 years.

Are you given x and asked for R(x) **or** are you given R(x) and asked for x?

Do it on the calculator with parentheses around the whole bottom.