Whenever you get stuck, think back to how you deal with normal fractions like $\frac{2}{4}+\frac{1}{4}$.

Intermediate algebra
Class notes
Adding and Subtracting Rational Expressions with Like Denominators and Finding Least Common Denominators (section 14.3)

Least Common Denominators (LCDs) will be needed in the next section.

Recall fractions:

Add $\frac{2}{6}+\frac{1}{6}$.

Add the tops, leave the bottom as is. Simplify if needed.

We will add and subtract rational expressions with like denominators (bottoms) the same way. There is one more detail to consider when subtracting which we will talk about later.

expl 1: Add.

$$\frac{3x}{4x+28}+\frac{21}{4x+28}$$

Add the tops, leave the bottom as is.

Factor top and bottom and look for common factors to cancel.

expl 2: Add.

$$\frac{x+14}{x^2+7x+6}+\frac{3x+10}{x^2+7x+6}$$

expl 3: Subtract.

$$\frac{5x-23}{x-9}-\frac{3x-5}{x-9}$$

Make sure the **whole** 3x – 5 gets subtracted. What do you need to do?

Remember, when you subtract more than one term, like 3x – 5, you need parentheses around it. Did you get 2 as your final answer?

We will see addition and subtraction problems in the next section which require this.

Least Common Denominators:

To add or subtract fractions, we need them to have the same denominator. But what if they don't, like $\frac{1}{6}+\frac{2}{9}$?

We need to change one or both bottoms so that they match.

Add $\frac{1}{6}+\frac{2}{9}$.

The most efficient way to get the same denominator is the LCD (least common denominator). We want to change both denominators to a number that is a multiple of both 6 and 9, preferably the smallest common multiple. Write out the first several multiples of 6 and 9 below. I have started the lists for you.

6: 6, 12, _____, _____, _____, …

9: 9, 18, _____, _____, _____, …

Circle the least common multiple.

So, we want to change both denominators to 18. But how? And it's important that as we rewrite our fractions, we do not actually change their values, just their appearance.

$$\frac{1}{6}+\frac{2}{9}$$

We want both bottoms to be 18. What could we do to $1/6$ to get 18 on bottom without changing its value?

Notice $\frac{1}{6}\cdot\frac{3}{3}=\frac{3}{18}$

Why does multiplying by $3/3$ not change the value of our original fraction?

So, how would you add $\frac{1}{6}+\frac{2}{9}$? Do it now.

Did you get $\frac{7}{18}$?

Let's do another. Add $\frac{4}{5}+\frac{3}{15}$.

Do you need to change both denominators?

Did you get 1? Hee hee, cool, huh?

Let's move to algebra. We will focus on finding LCDs here and move on to adding and subtracting in the next section.

expl 4: Find the LCD of $\frac{7}{6x^2}$ and $\frac{9}{8x^3}$.

Find the LCD of 6 and 8.

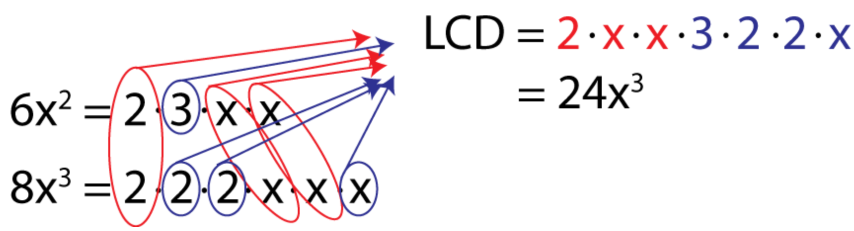
6: 6, 12, 18, …
8: 8, 16, 24, …

Find the LCD of x^2 and x^3.

x^2: $x^2, x^3, x^4, \ldots$
x^3: $x^3, x^4, x^5, \ldots$

Alternate method to find LCD:

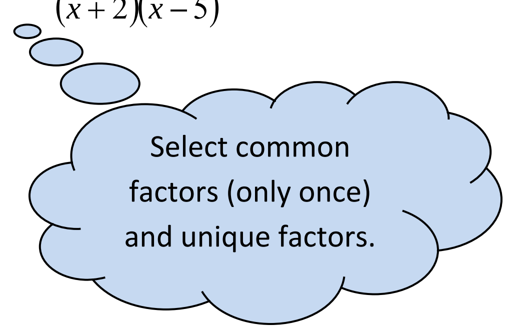
Factor each denominator completely. The LCD will always be the product of each common factor (only once) and all unique factors. See how we find the LCD of $6x^2$ and $8x^3$ below.



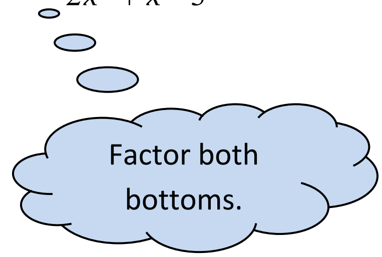
expl 5: Use the alternate method to find the LCD of $\frac{5}{12x^4}$ and $\frac{7}{8x^2}$.

expl 6: Use whichever method you prefer to find the LCD of $\frac{3}{5x^2}$ and $\frac{5}{6x}$.

expl 7: Use the alternate method to find the LCD of $\frac{3}{(x+2)(x+3)}$ and $\frac{4x}{(x+2)(x-5)}$.



expl 8: Use whichever method you prefer to find the LCD of $\frac{8}{x^2+4x-5}$ and $\frac{3}{2x^2+x-3}$.



In the next section, we will continue finding LCDs in order to add and subtract these rational expressions.