We will use LCDs to simplify and solve equations containing these nasty fractions.

Intermediate algebra
Class notes
Solving Equations with Rational Expressions and Formulas (section 14.5)

We will play around with formulas like $A = l \cdot w$.

Solving an equation means to rewrite it over and over until the variable is isolated, like $x = 4$. That will be the number (or numbers) that makes the original equation true. There are lots of different ways to do this depending on the type of equation. You know methods for linear equations (such as $3x + 4 = 10$) and quadratic equations (such as $x^2 - 7x - 60 = 0$). Here we learn how best to deal with rational equations like $\frac{x}{5} + \frac{1}{6} = \frac{8}{3}$ or $\frac{6x+13}{x-1} = \frac{4x+15}{x-1}$.

Main idea: The most efficient method is to multiply all fractions by their LCD. That eliminates the fractions as you will see. Make sure you always multiply **every term** by the LCD.

expl 1: Solve $\frac{x}{5} + \frac{1}{6} = \frac{8}{3}$.

What is the LCD of 5, 6, and 3?

Multiply every fraction by 30.

When you multiplied by 30, did the denominators in your fractions get eliminated?

$30 \cdot \frac{x}{5} = ?$

$30 \cdot \frac{1}{6} = ?$

$30 \cdot \frac{8}{3} = ?$

So our equation became $6x + 5 = 80$. Solve that as you would any linear equation.

Notice how each step gets you closer to having x alone. That is the whole idea of solving an equation.

Checking your answer:

A very important step in solving rational equations is checking your answer. It turns out that sometimes a solution actually turns out to be what is called an "extraneous solution". Even though you get it by doing good algebra, it turns out that it **does not make the equation true** after all. Remember, the whole point is to find the value(s) of the variable that make the equation true.

Start with the original equation and see if 12.5 really is a solution. Does it make $\frac{x}{5}+\frac{1}{6}=\frac{8}{3}$ true?

I would personally do this on my calculator. Enter the following into your calculator.

Left side of equation: 12.5 / 5 + 1 / 6 $\boxed{\text{ENTER}}$

Right side of equation: 8 / 3 $\boxed{\text{ENTER}}$

Use division key for fractions.

Do they match? Does 12.5 make the equation true?

expl 2: Solve.

$$\frac{x-3}{x+9}=\frac{6}{x+2}$$

What is the LCD here? So multiply both fractions by that.

How does multiplying the fractions by (x + 9)(x + 2) get rid of the fractions?

Your equation will become quadratic. How do you solve those?

Check your answers!

Cross-multiplying:

Notice the last problem quickly became (x + 2)(x – 3) = 6(x + 9) once we multiplied by the LCD. We were then able to solve it like any quadratic equation. That step can be short-cut by what is commonly called cross-multiplying. When you have an equation with just one fraction on each side, you can "cross-multiply" to get a simpler equation with no fractions. But beware! It only works if the equation is in the form "one fraction = another fraction". Many times it saves a little brain power. But sometimes it actually creates more work like this next example.

expl 3: Solve.

$$\frac{6x+13}{x-1}=\frac{4x+15}{x-1}$$

What is the LCD here? So multiply both fractions by that.

What happens when you multiply both sides by (x – 1)?

Cross-multiplying actually makes more work for us. How?

Check your answer. What happens when you substitute it back into the original equation? This solution (you should have gotten x = 1, did you?) is called an **extraneous solution**. We got it through perfectly good algebra but it really does **not make the equation true**. Bummer! So what do we say the solution to the equation is?

Why? What happened?

expl 4: Solve. Check your answer.

$$\frac{3x}{4}+\frac{5}{6}=\frac{x}{12}+\frac{7}{8}$$

Multiply all fractions by the LCD.

Factor all bottoms to find the LCD.

expl 5: Solve. Check your answer.

$$\frac{x-2}{x^2+13x+40}+\frac{x+5}{x^2+7x-8}=\frac{x+3}{x^2+4x-5}$$

Be sure to check your answers in the **original** equation.

Formulas: We will solve these formulas by isolating a variable that is on the right side of the equation. To understand what to do, imagine how you would solve the equation with numbers plugged in for the other variables.

expl 6: Solve P = 2l + 2w for w.

For #6, how would you solve 4 = 3 + 2w?

expl 7: Solve $W=\frac{CE^2}{2}$ for C.

How would you solve $5=\frac{3C}{2}$?