Read the problem twice. Focus on what you are asked to find. Think about what is equal to what. Translate to an equation and solve. Check.

Intermediate algebra
Class notes
Problem Solving with Rational Equations and Proportions (section 14.6)

We will do many examples here. Remember to use algebra to express each part of the story problem. Then think about how the parts are related to form the equation. Remember to always define your variable when you write an equation. Specifically write down what you are letting the x represent. This gets much more important as the problems get more complicated as you will see in this section.

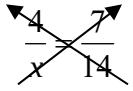
Definition: Ratio: A comparison of two quantities, usually written in fraction form when we use them in algebra.

expls: 50 calories to 17 grams, $\frac{50\ calories}{17\ grams}$, 50:17, 30 pounds to 4 iguanas, 30:4, 30/4

Definition: Proportion: an equation in the form of "ratio = ratio". I like to think of it as "one fraction equals another fraction".

expls: $\frac{x-3}{x+9}=\frac{6}{x+2}$, $\frac{6x+13}{x-1}=\frac{4x+15}{x-1}$, $\frac{4}{x}=\frac{7}{14}$

We will most often start solving these by cross-multiplying, shown in previous section. Cross-multiplying is briefly reviewed below.



$$4\cdot 14 = 7x$$
$$56 = 7x$$
$$8 = x$$

In story problems, proportions are used when we know the ratio of two numbers is the same as the ratio of two other numbers. Usually we are given three of the numbers and are asked to find the fourth number. We will make an equation and then solve it.

expl 1: The ratio of the weight of an object on Earth to the weight of that object on Pluto is 100 to 3. If an elephant weighs 4000 pounds on Earth, how much would it weigh on Pluto?

The order they state the ratio tells you a lot.

$$\frac{100}{3} = \frac{any\ object's\ weight\ on\ Earth}{that\ object's\ weight\ on\ Pluto}$$

Earth on top, Pluto on bottom!

Let x represent the weight of the elephant on Pluto since that is what we want to know.

Now set up a proportion:
given ratio = specific ratio in problem

$$\frac{100}{3} = \frac{elephant's\ weight\ on\ Earth}{elephant's\ weight\ on\ Pluto\ (x)}$$

The thing to remember about proportions is that **what is on top on one side should be on top on the other side of the equation**. The previous problem could **not** be set up as $\frac{100}{3} = \frac{x}{4000}$.

Since x is the weight on Pluto, it goes on bottom across from the 3, which is Pluto's part of the given ratio. Solve the equation from above, attach units to your answer, and make sure it makes sense. Write your answer in phrase or sentence form.

expl 2: There are 150 calories in one slice (33 grams) of banana bread. If I measure out 50 grams of banana bread, how many calories are in it?

What are you asked to find?
Define your variable to be that.
Be specific and write it down.

Each ratio can be thought of as "calories to grams" or $\frac{calories}{grams}$.

Set up a proportion:
given ratio = specific ratio in problem
and solve!

expl 3: Solve.

$$\frac{x+1}{2x+3} = \frac{2}{3}$$

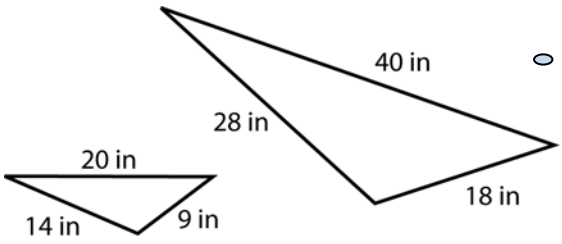
Cross-multiply to get rid of the fractions. Then isolate the variable.

Similar triangles:

If two triangles are similar, the ratios of their corresponding sides are equal. That will help us solve for unknown sides.

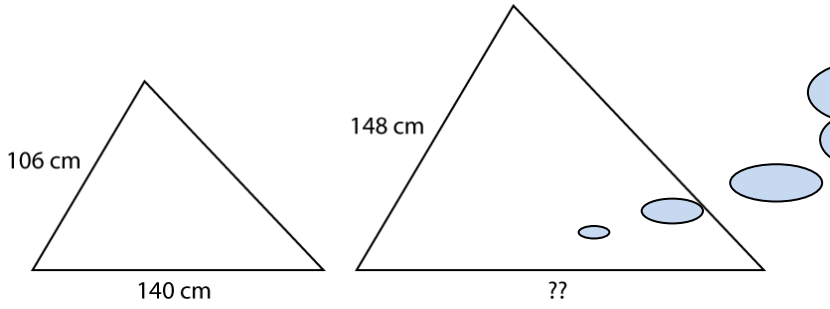
Definition: Similar triangles: Two triangles are similar if they have the same shape. One may be larger than the other or be in a different orientation but they will have the same shape.

expl:



$$\frac{20}{40} = \frac{14}{28} = \frac{9}{18}$$

expl 4: Find the unknown length for the pair of similar triangles below.



Use x for the unknown side. Set up a proportion and solve it.

Reciprocals:

What do we mean by a reciprocal? Find the following reciprocals.

The reciprocal of $\frac{2}{3}$ is ?

"flip the fraction"

The reciprocal of 4 is ?

The reciprocal of x is ?

expl 5: One positive number is six less than another positive number. If two times the reciprocal of the smaller number is added to five times the reciprocal of the larger number, the sum is equal to 1. Find the two numbers.

Let x = one of the numbers

?? = other number

Use "one positive number is six less than another positive number" to determine an expression for the other number.

Now determine which of your two numbers is smaller.

Go step by step.

What are the reciprocals of your numbers?

What is "two times the reciprocal of the smaller number"?

What is "five times the reciprocal of the larger number"?

Their sum is 1. Use that to form an equation.

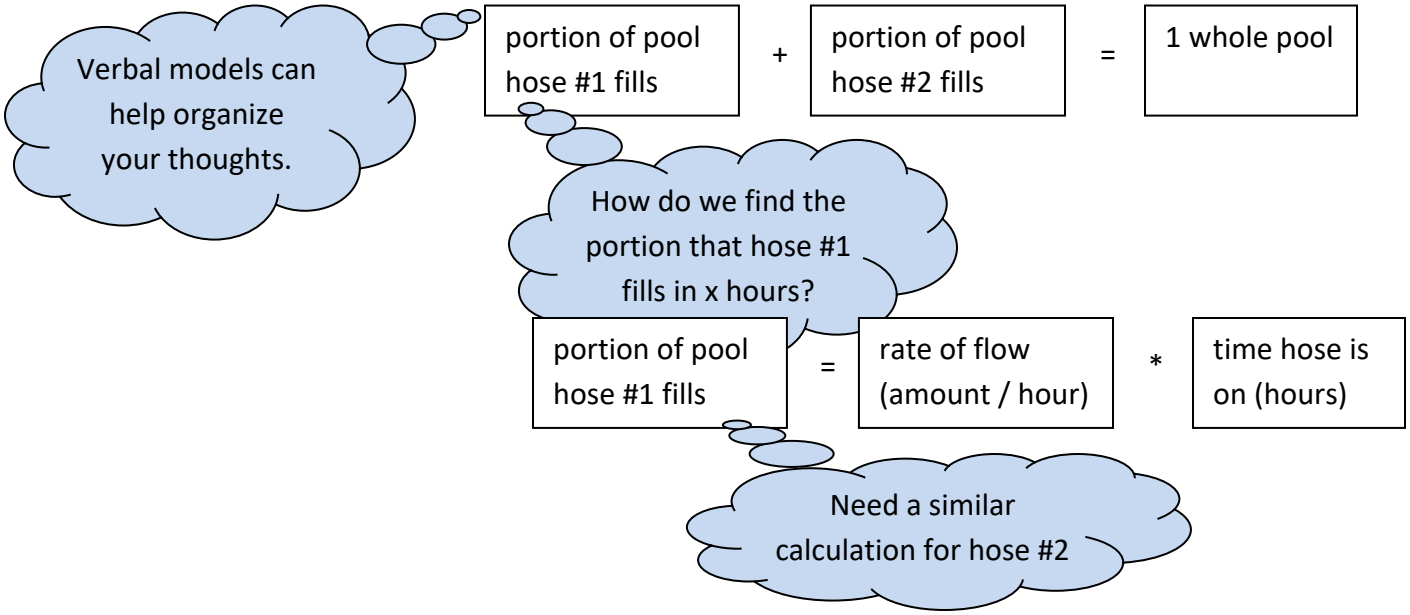
Solve the equation to find the two numbers. Remember how we did this in the previous section.

Check your answers with the original set-up. Do they fit?

Work-rate problems:

expl 6: A hose can fill a 40,000 gallon swimming pool in 18 hours. A neighbor's hose can fill it in 24 hours. If the hoses run at the same time, how long will it take to fill the pool?

Let x be what we are asked to find, the number of hours both hoses will run.



One major point to realize is that the hoses fill evenly over the whole time. If it takes 18 hours for hose #1 to fill the pool, then how much of the pool (what fraction) is filled in 1 hour? Likewise, how much of the pool is filled by hose #2 in 1 hour? This gives us the rates of flow for the second verbal model above.

Now form an equation by combining the two verbal models above and then solve. Round your answer to two decimal places (hundredths place).

Distance problems:

$d = r \cdot t$

d = distance (feet, miles, etc.)

r = rate (feet per second, miles per hour, etc.)

t = time (seconds, hours, etc.)

expl 7: Denae is swimming in a river that flows downstream at a speed of .5 meters per second. It takes her the same amount of time to swim 500 meters upstream (against current) as it does to swim 1000 meters downstream (with current). Find Denae's swimming speed (in meters per second) in still water.

Let x represent her swimming speed in still water.

?? = Denae's downstream swimming speed (with current)

?? = Denae's upstream swimming speed (against current)

She is pushed along by the current or slowed down by it.

What two things are we told are equal?

time to swim downstream	=	time to swim upstream

$d = r \cdot t$

Time equals distance divided by rate. Fill in what you know and solve.

How you set up an equation will depend a lot on the problem itself. Do not try to follow examples too closely. Try to think through the problems rather than memorizing procedures.