We will earn interest *not only* on the original deposit but also on *previously* earned interest.

Business Mathematics
Class notes
Compound Interest (section 10.1)

Main idea:

In the previous chapter, we studied **simple interest**.

Consider a principal of \$1,000 invested at 5% *simple interest* for 4 years. The money earns interest at the rate of 5% of the initial deposit every year for 4 years. Hence, $I = PRT = 1000 \times .05 \times 4 =$ \$200. After four years, they have a total of \$1,200.

With simple interest, interest is based on how much was initially deposited. But after one year, we have an extra \$50 (the interest earned or 5% of \$1,000) that we could invest. *If* we put that money back into the account and let it earn interest, we call that **compound interest**. All interest that is earned is subsequently reinvested to start earning the same rate as the initial deposit.

We will have formulas with new variables and a table or two or… lots. Really, lots of tables, we are going to see a lot of tables, people.

We will use a worksheet to see how compound interest plays out using simple interest formulas to better your understanding.

Let's look at some of the terminology and variables we will be using. We have seen these variables but we have a few new terms.

R = annual interest rate
P = principal or initial investment
M = future value, maturity value, or **compound amount**
I = interest earned
T = time in years

T will *not* appear specifically in the formula but it is needed.

Definition: Compounding: when interest is calculated and it is added to the account's balance.

Definition: Compounding period: the amount of time between each compounding.

Accounts are most often **compounded quarterly** (four times a year), **monthly**, **daily** (365 times a year), or **semiannually** (twice a year).

Worksheet: Understanding Compound Interest Through Simple Interest:
This worksheet will work us through an example where we find compound interest by using the simple interest formula again and again. We will also compare compound and simple interest investments to see which has a better yield. Lastly, we get a taste of the compound interest formula.

Compound Interest Formula:
As the worksheet introduces, the formula below calculates the money in a compound interest-bearing account.

$$M = P\,(1 + i)^{n}$$

M = **maturity value** (future amount, compound amount)
P = **principal** (initial deposit, investment)
i = **interest rate per compounding period**
n = **total number of compounding periods**

This i is *not* the same as $I = PRT$.

Let's talk about those new variables, i and n.

First, there is ***i* = interest rate per compounding period.** The interest rate in problems is usually stated as an *annual* interest rate. We need to divide that by the number of times it compounds in a year to get i.

expl 1a: An account pays 8% compounded quarterly. What is i, the interest rate *per compounding period*?

Second, we have this ***n*** = **total number of compounding periods.** This means how many times the account will compound (that is, calculate and add in the interest) during the *whole* time it is invested. So we multiply the number of years (T) by the number of times it compounds per year to get n.

expl 1b: Let's say this account is compounded quarterly for five years. How many times *total* will it compound? In other words, what is n?

Compound Interest Formula:

We also have the familiar $M = P + I$, but since we find M in the formula on the previous page, a more useful version is below.

$I = M - P$

Maturity value equals principal plus interest.

Both i and I have meaning, and the meanings are different!

Also, if they say the account is compounded…	the number of times a year is …
annually	1
semiannually	2
quarterly	4
monthly	12
daily	365

expl 2a: Find the compound amount and interest for an investment of \$20,000 at 5% compounded quarterly for ¾ of a year.

Be careful with i and n.

expl 2b: Write a sentence to describe what we found.

Compound Interest Table:

So that is all well and good but… You may see a table that basically does the computation of the $(1 + i)^n$ part of the formula for you. It is copied on the next page and we will use it on the next example.

expl 3: Use the table to find the compound amount and interest for the following investment. Follow the steps outlined.

$12,300 at 3% compounded semiannually for 4 years

a.) Find i, the interest rate per compounding period.

			Compound Inter			
			Interest Rate per Comp			
Period	$\frac{1}{2}$%	1%	$1\frac{1}{2}$%	2%	$2\frac{1}{2}$%	
1	1.00500	1.01000	1.01500	1.02000	1.02500	1.
2	1.01003	1.02010	1.03023	1.04040	1.05063	1.
3	1.01508	1.03030	1.04568	1.06121	1.07689	1.
4	1.02015	1.04060	1.06136	1.08243	1.10381	1.
5	1.02525	1.05101	1.07728	1.10408	1.13141	1.
6	1.03038	1.06152	1.09344	1.12616	1.15969	1.
7	1.03553	1.07214	1.10984	1.14869	1.18869	1.
8	1.04071	1.08286	1.12649	1.17166	1.21840	1.
9	1.04591	1.09369	1.14339	[illegible]	1.24886	1.
10	1.05114	1.10462	1.16054	[illegible]	[illegible]	[illegible]
11	1.05640	1.11567	1.17795	[illegible]	[illegible]	[illegible]

b.) Find n, the total number of times it will compound.

Circle the multiplier.

c.) Look up the value of i along the top of the table and the value of n along either outside column. The intersection of the row and column is a multiplier. Multiply it by the principal to find the compound amount.

This tells us that $1 invested this way will become $1.12649.

d.) Lastly, subtract to find the interest that the investment earned.

Using the formula versus the table yields final answers that round to different values.

Compound Interest Table:
Maturity Value for $1 in Principal:

Compound Interest Table											
Interest Rate per Compounding Period											
Period	$\frac{1}{2}$%	1%	$1\frac{1}{2}$%	2%	$2\frac{1}{2}$%	3%	4%	5%	6%	8%	Period
1	1.00500	1.01000	1.01500	1.02000	1.02500	1.03000	1.04000	1.05000	1.06000	1.08000	1
2	1.01003	1.02010	1.03023	1.04040	1.05063	1.06090	1.08160	1.10250	1.12360	1.16640	2
3	1.01508	1.03030	1.04568	1.06121	1.07689	1.09273	1.12486	1.15763	1.19102	1.25971	3
4	1.02015	1.04060	1.06136	1.08243	1.10381	1.12551	1.16986	1.21551	1.26248	1.36049	4
5	1.02525	1.05101	1.07728	1.10408	1.13141	1.15927	1.21665	1.27628	1.33823	1.46933	5
6	1.03038	1.06152	1.09344	1.12616	1.15969	1.19405	1.26532	1.34010	1.41852	1.58687	6
7	1.03553	1.07214	1.10984	1.14869	1.18869	1.22987	1.31593	1.40710	1.50363	1.71382	7
8	1.04071	1.08286	1.12649	1.17166	1.21840	1.26677	1.36857	1.47746	1.59385	1.85093	8
9	1.04591	1.09369	1.14339	1.19509	1.24886	1.30477	1.42331	1.55133	1.68948	1.99900	9
10	1.05114	1.10462	1.16054	1.21899	1.28008	1.34392	1.48024	1.62889	1.79085	2.15892	10
11	1.05640	1.11567	1.17795	1.24337	1.31209	1.38423	1.53945	1.71034	1.89830	2.33164	11
12	1.06168	1.12683	1.19562	1.26824	1.34489	1.42576	1.60103	1.79586	2.01220	2.51817	12
13	1.06699	1.13809	1.21355	1.29361	1.37851	1.46853	1.66507	1.88565	2.13293	2.71962	13
14	1.07232	1.14947	1.23176	1.31948	1.41297	1.51259	1.73168	1.97993	2.26090	2.93719	14
15	1.07768	1.16097	1.25023	1.34587	1.44830	1.55797	1.80094	2.07893	2.39656	3.17217	15
16	1.08307	1.17258	1.26899	1.37279	1.48451	1.60471	1.87298	2.18287	2.54035	3.42594	16
17	1.08849	1.18430	1.28802	1.40024	1.52162	1.65285	1.94790	2.29202	2.69277	3.70002	17
18	1.09393	1.19615	1.30734	1.42825	1.55966	1.70243	2.02582	2.40662	2.85434	3.99602	18
19	1.09940	1.20811	1.32695	1.45681	1.59865	1.75351	2.10685	2.52695	3.02560	4.31570	19
20	1.10490	1.22019	1.34686	1.48595	1.63862	1.80611	2.19112	2.65330	3.20714	4.66096	20
21	1.11042	1.23239	1.36706	1.51567	1.67958	1.86029	2.27877	2.78596	3.39956	5.03383	21
22	1.11597	1.24472	1.38756	1.54598	1.72157	1.91610	2.36992	2.92526	3.60354	5.43654	22
23	1.12155	1.25716	1.40838	1.57690	1.76461	1.97359	2.46472	3.07152	3.81975	5.87146	23
24	1.12716	1.26973	1.42950	1.60844	1.80873	2.03279	2.56330	3.22510	4.04893	6.34118	24
25	1.13280	1.28243	1.45095	1.64061	1.85394	2.09378	2.66584	3.38635	4.29187	6.84848	25
26	1.13846	1.29526	1.47271	1.67342	1.90029	2.15659	2.77247	3.55567	4.54938	7.39635	26
27	1.14415	1.30821	1.49480	1.70689	1.94780	2.22129	2.88337	3.73346	4.82235	7.98806	27
28	1.14987	1.32129	1.51722	1.74102	1.99650	2.28793	2.99870	3.92013	5.11169	8.62711	28
29	1.15562	1.33450	1.53998	1.77584	2.04641	2.35657	3.11865	4.11614	5.41839	9.31727	29
30	1.16140	1.34785	1.56308	1.81136	2.09757	2.42726	3.24340	4.32194	5.74349	10.06266	30

(source: Business Math, Clendenen and Salzman, 14^{th} ed.)

This table calculates for us the value of $(1+i)^n$ where i is the interest rate per compounding period (listed along the top of the table) and n is the total number of compounding periods (listed down either the right-most or left-most column).

The values given in the body of the table tell us the maturity value for each $1 in principal. We use this value in the calculation of the maturity value for compound interest accounts and loans.