As before, we use a table to find a multiplier, which simplifies our formula. We will also estimate a business' worth.

Business Mathematics
Class notes
Present Value and Future Value (section 10.3)

We will be finding the principal needed in order to accumulate a desired amount of money in the future. In previous sections, we went the other way.

Definition: Future value: the maturity value of a bank account or the amount in the account at the end of the investment period. In the compound interest formula, this is M.

Definition: Present value: the initial investment, deposit, or principal that will grow (with interest) to the amount desired. In the compound interest formula, this is P.

Again, our formula is the familiar $M = P(1+i)^n$. However, we will solve this for P, getting us $P = \dfrac{M}{(1+i)^n}$ or rather, $P = M \cdot \dfrac{1}{(1+i)^n}$.

We will be looking up the value of $\dfrac{1}{(1+i)^n}$ in a table and simply multiplying that with our M to figure P. So, recall that i is the interest rate *per* compounding period and n is the total number of times the account compounds.

We will also use this process to estimate a business' worth. A business might want this if they want to sell the business.

expl 1: Find the present value and interest earned for the following desired maturity value and terms.
Amount needed: $269,000
Time (years): 5 years
Interest rate: 8%
Compounded: semiannually

Remember the "amount needed" is M. We seek P.

Figure out what i and n must be. Look them up on the table at the back of the notes.

Multiply the table entry by M and we get P.

So, if we invest ___________________ now, it will grow to $269,000 in five years.

Estimating the Value of a Business:

expl 2: John figures his bike shop is worth $88,000 if sold today and that it will grow in value at 8% per year compounded annually for the next six years. If he sells the business, the funds will be invested at 5% compounded semiannually. Round to the nearest dollar.

a.) Find the future value of his shop.

Treat this $88,000 as an investment that compounds annually at 8% for 6 years. Use the Compound Interest Table repeated on page 4.

b.) What price should he insist on at this time if he sells?

What P would he have to invest now at 5% compounded semiannually (for 6 years) so that he would end up with the equivalent of the future value of his shop?

So, he should sell his bike shop for _________________. He will then invest it and earn the equivalent of the future value of his bike shop which is ____________________.

Problems in the homework may *not* explicitly use this situation. However, if you are asked to find the future value of an investment and then find the present value needed to accumulate that amount in another investment, follow the steps that we did here.

Present Value of a Dollar Table

	Interest Rate per Compounding Period									
# of Periods	**1%**	**1.50%**	**2%**	**2.50%**	**3%**	**4%**	**5%**	**6%**	**8%**	**10%**
1	0.99010	0.98522	0.98039	0.97561	0.97087	0.96154	0.95238	0.94340	0.92593	0.90909
2	0.98030	0.97066	0.96117	0.95181	0.94260	0.92456	0.90703	0.89000	0.85734	0.82645
3	0.97059	0.95632	0.94232	0.92860	0.91514	0.88900	0.86384	0.83962	0.79383	0.75131
4	0.96098	0.94218	0.92385	0.90595	0.88849	0.85480	0.82270	0.79209	0.73503	0.68301
5	0.95147	0.92826	0.90573	0.88385	0.86261	0.82193	0.78353	0.74726	0.68058	0.62092
6	0.94205	0.91454	0.88797	0.86230	0.83748	0.79031	0.74622	0.70496	0.63017	0.56447
7	0.93272	0.90103	0.87056	0.84127	0.81309	0.75992	0.71068	0.66506	0.58349	0.51316
8	0.92348	0.88771	0.85349	0.82075	0.78941	0.73069	0.67684	0.62741	0.54027	0.46651
9	0.91434	0.87459	0.83676	0.80073	0.76642	0.70259	0.64461	0.59190	0.50025	0.42410
10	0.90529	0.86167	0.82035	0.78120	0.74409	0.67556	0.61391	0.55839	0.46319	0.38554
11	0.89632	0.84893	0.80426	0.76214	0.72242	0.64958	0.58468	0.52679	0.42888	0.35049
12	0.88745	0.83639	0.78849	0.74356	0.70138	0.62460	0.55684	0.49697	0.39711	0.31863
13	0.87866	0.82403	0.77303	0.72542	0.68095	0.60057	0.53032	0.46884	0.36770	0.28966
14	0.86996	0.81185	0.75788	0.70773	0.66112	0.57748	0.50507	0.44230	0.34046	0.26333
15	0.86135	0.79985	0.74301	0.69047	0.64186	0.55526	0.48102	0.41727	0.31524	0.23939
16	0.85282	0.78803	0.72845	0.67362	0.62317	0.53391	0.45811	0.39365	0.29189	0.21763
17	0.84438	0.77639	0.71416	0.65720	0.60502	0.51337	0.43630	0.37136	0.27027	0.19784
18	0.83602	0.76491	0.70016	0.64117	0.58739	0.49363	0.41552	0.35034	0.25025	0.17986
19	0.82774	0.75361	0.68643	0.62553	0.57029	0.47464	0.39573	0.33051	0.23171	0.16351
20	0.81954	0.74247	0.67297	0.61027	0.55368	0.45639	0.37689	0.31180	0.21455	0.14864
21	0.81143	0.73150	0.65978	0.59539	0.53755	0.43883	0.35894	0.29416	0.19866	0.13513
22	0.80340	0.72069	0.64684	0.58086	0.52189	0.42196	0.34185	0.27751	0.18394	0.12285
23	0.79544	0.71004	0.63416	0.56670	0.50669	0.40573	0.32557	0.26180	0.17032	0.11168
24	0.78757	0.69954	0.62172	0.55288	0.49193	0.39012	0.31007	0.24698	0.15770	0.10153
25	0.77977	0.68921	0.60953	0.53939	0.47761	0.37512	0.29530	0.23300	0.14602	0.09230
26	0.77205	0.67902	0.59758	0.52623	0.46369	0.36069	0.28124	0.21981	0.13520	0.08391
27	0.76440	0.66899	0.58586	0.51340	0.45019	0.34682	0.26785	0.20737	0.12519	0.07628
28	0.75684	0.65910	0.57437	0.50088	0.43708	0.33348	0.25509	0.19563	0.11591	0.06934
29	0.74934	0.64936	0.56311	0.48866	0.42435	0.32065	0.24295	0.18456	0.10733	0.06304
30	0.74192	0.63976	0.55207	0.47674	0.41199	0.30832	0.23138	0.17411	0.09938	0.05731
31	0.73458	0.63031	0.54125	0.46511	0.39999	0.29646	0.22036	0.16425	0.09202	0.05210
32	0.72730	0.62099	0.53063	0.45377	0.38834	0.28506	0.20987	0.15496	0.08520	0.04736
33	0.72010	0.61182	0.52023	0.44270	0.37703	0.27409	0.19987	0.14619	0.07889	0.04306
34	0.71297	0.60277	0.51003	0.43191	0.36604	0.26355	0.19035	0.13791	0.07305	0.03914
35	0.70591	0.59387	0.50003	0.42137	0.35538	0.25342	0.18129	0.13011	0.06763	0.03558
36	0.69892	0.58509	0.49022	0.41109	0.34503	0.24367	0.17266	0.12274	0.06262	0.03235
37	0.69200	0.57644	0.48061	0.40107	0.33498	0.23430	0.16444	0.11579	0.05799	0.02941

Table entries are $1 / (1 + i)^n$. Multiply future value (M) by table entry to get present value (P).

Compound Interest Table:
Maturity Value for $1 in Principal:

This is from section 10.1.

Compound Interest Table											
	Interest Rate per Compounding Period										
Period	$\frac{1}{2}$%	1%	$1\frac{1}{2}$%	2%	$2\frac{1}{2}$%	3%	4%	5%	6%	8%	Period
1	1.00500	1.01000	1.01500	1.02000	1.02500	1.03000	1.04000	1.05000	1.06000	1.08000	1
2	1.01003	1.02010	1.03023	1.04040	1.05063	1.06090	1.08160	1.10250	1.12360	1.16640	2
3	1.01508	1.03030	1.04568	1.06121	1.07689	1.09273	1.12486	1.15763	1.19102	1.25971	3
4	1.02015	1.04060	1.06136	1.08243	1.10381	1.12551	1.16986	1.21551	1.26248	1.36049	4
5	1.02525	1.05101	1.07728	1.10408	1.13141	1.15927	1.21665	1.27628	1.33823	1.46933	5
6	1.03038	1.06152	1.09344	1.12616	1.15969	1.19405	1.26532	1.34010	1.41852	1.58687	6
7	1.03553	1.07214	1.10984	1.14869	1.18869	1.22987	1.31593	1.40710	1.50363	1.71382	7
8	1.04071	1.08286	1.12649	1.17166	1.21840	1.26677	1.36857	1.47746	1.59385	1.85093	8
9	1.04591	1.09369	1.14339	1.19509	1.24886	1.30477	1.42331	1.55133	1.68948	1.99900	9
10	1.05114	1.10462	1.16054	1.21899	1.28008	1.34392	1.48024	1.62889	1.79085	2.15892	10
11	1.05640	1.11567	1.17795	1.24337	1.31209	1.38423	1.53945	1.71034	1.89830	2.33164	11
12	1.06168	1.12683	1.19562	1.26824	1.34489	1.42576	1.60103	1.79586	2.01220	2.51817	12
13	1.06699	1.13809	1.21355	1.29361	1.37851	1.46853	1.66507	1.88565	2.13293	2.71962	13
14	1.07232	1.14947	1.23176	1.31948	1.41297	1.51259	1.73168	1.97993	2.26090	2.93719	14
15	1.07768	1.16097	1.25023	1.34587	1.44830	1.55797	1.80094	2.07893	2.39656	3.17217	15
16	1.08307	1.17258	1.26899	1.37279	1.48451	1.60471	1.87298	2.18287	2.54035	3.42594	16
17	1.08849	1.18430	1.28802	1.40024	1.52162	1.65285	1.94790	2.29202	2.69277	3.70002	17
18	1.09393	1.19615	1.30734	1.42825	1.55966	1.70243	2.02582	2.40662	2.85434	3.99602	18
19	1.09940	1.20811	1.32695	1.45681	1.59865	1.75351	2.10685	2.52695	3.02560	4.31570	19
20	1.10490	1.22019	1.34686	1.48595	1.63862	1.80611	2.19112	2.65330	3.20714	4.66096	20
21	1.11042	1.23239	1.36706	1.51567	1.67958	1.86029	2.27877	2.78596	3.39956	5.03383	21
22	1.11597	1.24472	1.38756	1.54598	1.72157	1.91610	2.36992	2.92526	3.60354	5.43654	22
23	1.12155	1.25716	1.40838	1.57690	1.76461	1.97359	2.46472	3.07152	3.81975	5.87146	23
24	1.12716	1.26973	1.42950	1.60844	1.80873	2.03279	2.56330	3.22510	4.04893	6.34118	24
25	1.13280	1.28243	1.45095	1.64061	1.85394	2.09378	2.66584	3.38635	4.29187	6.84848	25
26	1.13846	1.29526	1.47271	1.67342	1.90029	2.15659	2.77247	3.55567	4.54938	7.39635	26
27	1.14415	1.30821	1.49480	1.70689	1.94780	2.22129	2.88337	3.73346	4.82235	7.98806	27
28	1.14987	1.32129	1.51722	1.74102	1.99650	2.28793	2.99870	3.92013	5.11169	8.62711	28
29	1.15562	1.33450	1.53998	1.77584	2.04641	2.35657	3.11865	4.11614	5.41839	9.31727	29
30	1.16140	1.34785	1.56308	1.81136	2.09757	2.42726	3.24340	4.32194	5.74349	10.06266	30

(source: Business Math, Clendenen and Salzman, 14th ed.)

This table calculates for us the value of $(1+i)^n$ where i is the interest rate per compounding period (listed along the top of the table) and n is the total number of compounding periods (listed down either the right-most or left-most column).

The values given in the body of the table tell us the maturity value for each $1 in principal. We use this value in the calculation of the maturity value for compound interest accounts and loans.