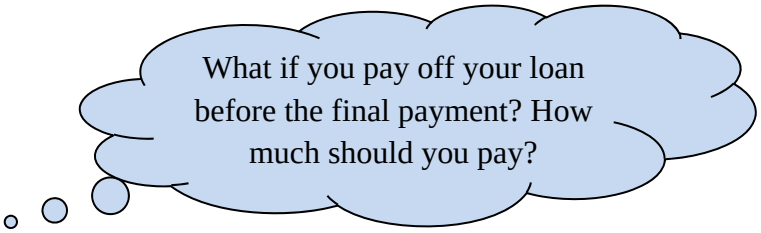



Business Mathematics
Class notes
Early Payoffs of Loans (section 12.3)

It is common to pay off part or all of a loan before it is due. The **Unites States Rule** requires that any loan payment be first applied to any interest owed. The balance of the payment is then used to reduce the principal of the loan.

We will, again, be using 360 days for one year.

Consider a loan that is to be paid off in equal, monthly payments for some time period. What happens when the borrower pays more than the monthly amount? How much is now due?

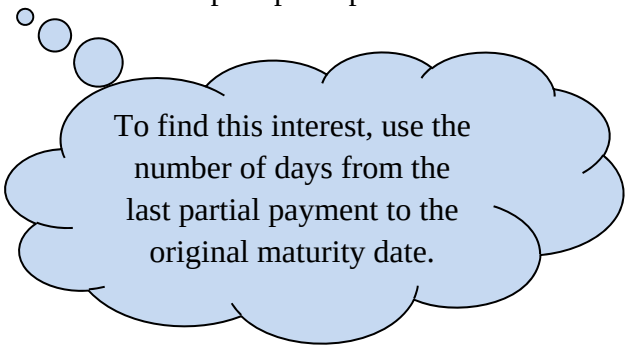
**Finding the Amount Due on the Maturity Date Using the United States Rule:**
1. Find the simple interest due from the date the loan was made until the partial payment was made. Use $I = PRT$.

2. Subtract this interest from the payment amount.

3. Any difference is used to reduce the principal. So subtract the difference from step 2 from the principal to find the new principal (balance).

4. Treat additional partial payments in the same way, always finding interest on *only the unpaid balance* after the previous partial payment. Also, use the number of days *between* payments to determine interest (for step 1).

5. The remaining principal plus interest on this unpaid principal is then due on the maturity date of the loan.




If you are asked to find the total interest paid, add the amounts you calculated in step 1.

expl 1: Find the balance due on the maturity date of this note. Find the total amount of interest paid on the note. Use the Unites States Rule.
Principal: $39,864
Interest: 9%
Time (days): 105
Partial payments: $8,458 on day 43 and $11,354 on day 88

Follow the first three steps for each payment.

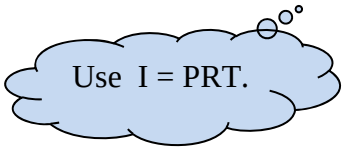
How much is the balance after the first payment? This balance holds from day 43 to day 88.

How much is the balance after the second payment? We are *not* done. Follow step 5.

They paid interest three times. Add to find the total interest paid.

**What does this mean to the borrower?**

Find the amount of interest they would have paid if they had *not* made early payments.




How much did they save in interest?

**Rule of 78:**
This is a rule that allows the lender to earn more of the interest in the early months of the loan. That way, if it is paid off early, the lender has already been made most of the interest.

The Rule of 78 gets its name from thinking of a loan with a 12 month period. Notice that
1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 = 78.

The payments are set up so the lender gets 12/78 of the total interest in the first month,
11/78 of the total interest in the second month,
10/78 of the total interest in the third month,
9/78 of the total interest in the fourth month, etc.

This continues until they would get 1/78 of the total interest in the final ($12^{th}$) month.

You can do this for any length of loan.

**Definition: Unearned interest:** The total finance charge (interest) on an installment loan is figured when the loan is made. Early payoff of a loan results in less interest being paid. The portion of the finance charge that has *not* been earned by the lender under the Rule of 78 is called the **unearned interest** or **refund**.

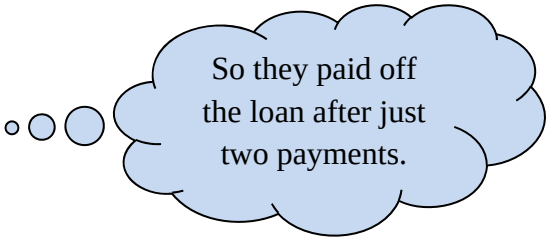
We do have a formula to find this amount.

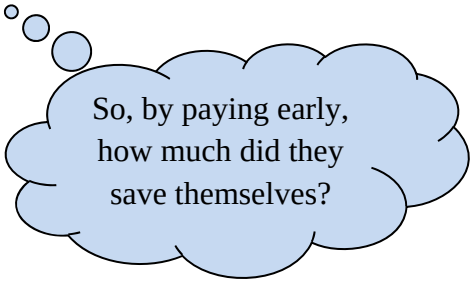
**Calculating Unearned Interest:**

Let U represent the unearned interest. Our formula is $U = F\left(\frac{N}{P}\right)\left(\frac{1+N}{1+P}\right)$.

Here, F is the finance charge, N is the number of payments *remaining*, and P is the original number of payments.

expl 2: This loan was paid in full before the maturity date. Find the amount of unearned interest. Use the Rule of 78.
Finance charge: $650
Total number of payments: 24
Remaining number of payments when paid in full: 22



expl 3: Movie 6, Inc. purchased movie projectors at a total cost of $12,200 with a down payment of $1,500. They agreed to make 12 monthly payments of $945 each to cover the remaining cost. Find the following.

a.) Find the finance charge. Label it F.

b.) Find the unearned interest if they pay the loan off early after the $8^{th}$ payment.

c.) Find the amount necessary to pay the loan in full (payoff amount) after the $8^{th}$ payment.

The finance charge will be the down payment *plus* the total paid *minus* the cost.

For parts *b* and *c*, assume they paid the first 8 payments on schedule and *then* paid off the loan.

For part *c*, the payoff amount is the total amount of remaining payments *minus* the unearned interest.