

We will use our regression skills again.

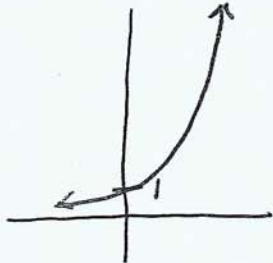
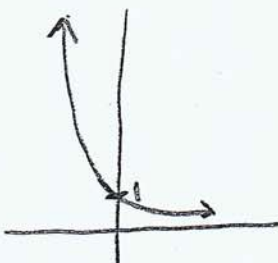
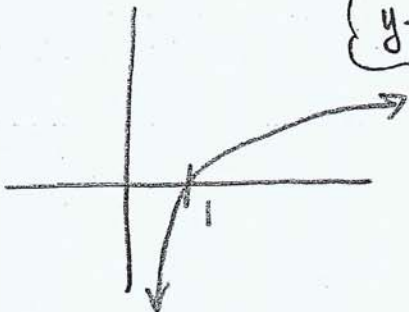
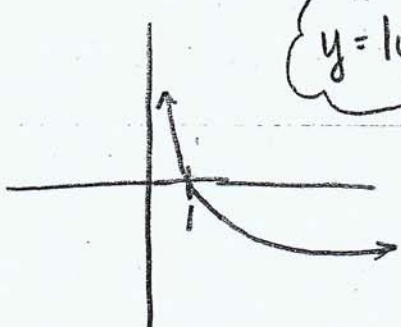
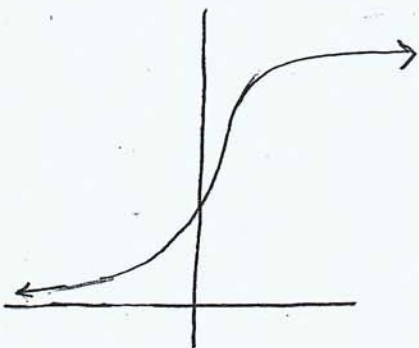
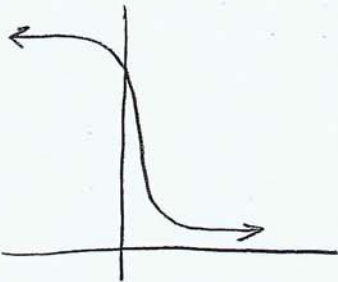
College algebra

Class notes

Exponential, Logarithmic, and Logistic Models from Data (section 6.9)

Recall the general forms that exponential and logarithmic graphs take. Do you remember the shape of the logistic models we just studied? If we collect data in the form of ordered pairs and plot them, do you think you would be able to tell the difference between an exponential or logistic shape? Let's try.

Give yourself a quick graph of each function type below. If you need, substitute values for the variables and use your calculator or refer to your Notes.

<p>Exponential $y = a^x$ when $a > 1$</p> 	<p>Exponential $y = a^x$ when $0 < a < 1$</p> 
<p>Logarithmic $y = \log_a x$ when $a > 1$</p> 	<p>Logarithmic $y = \log_a x$ when $0 < a < 1$</p> 
<p>Logistic $P(t) = \frac{c}{1 + ae^{-bt}}$ when $b > 0$</p> 	<p>Logistic $P(t) = \frac{c}{1 + ae^{-bt}}$ when $b < 0$</p> 

Correlation Coefficient, r :

Recall we used r a bit to determine if the line of best fit actually fit linear data well. We said that if this value was close to -1 or 1 , then it was a good fit. (Statisticians have a more exact process but we do *not* cover it here.)

It turns out that the correlation coefficient can be used for exponential and logarithmic regression as well. We will *not* use it for regression of logistic models.

Calculator Usage:

To enter data, you press the **STAT** button and select **Edit** (under **EDIT** menu) to enter the data into the columns **L1** and **L2** as before. To calculate the regression equations, press **STAT** once again but arrow over to **CALC**. In the list here, you will see **ExpReg** for exponential regression, **LnReg** for logarithmic regression, and **Logistic** for logistic regression.

We enter the independent (x) variable in **L1** and the dependent variable (y) in **L2**. Time is usually considered to be the independent variable. Pay attention to instructions given in the homework.

Round coefficients to four decimal places.

Use the **STATPLOT** editor to draw scatter plots as required. Of course, choose window settings that fit the data. Remember, most calculators will allow you to enter a regression equation into the **y-editor** (for **Y1**) by pressing **VARS** > **Statistics**. You then arrow over to **EQ** and select **RegEQ**.

Exponential Regression Equations:

expl 1: The data in the table below represent the annual revenue of Tesla, Inc. from 2010 to 2018. Answer the following questions.

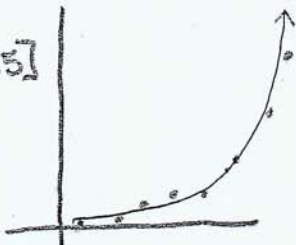
- a.) Use your calculator to draw a scatter plot using x to be the number of years since 2010.

Notice how the numbers in the table are increasing slowly at first and then very quickly.

Year	Revenue (\$ Billion)
2010 ($x = 0$)	0.12
2011 ($x = 1$)	0.20
2012 ($x = 2$)	0.41
2013 ($x = 3$)	2.01
2014 ($x = 4$)	3.20
2015 ($x = 5$)	4.05
2016 ($x = 6$)	7.00
2017 ($x = 7$)	11.76
2018 ($x = 8$)	21.46

- b.) Use your calculator to build an exponential regression equation. Graph this function on top of the scatter plot to check it visually. Does it fit the data?

Window
[0,10] x [0,25]



$$y = 0.143 * 1.92655^x$$

yes - it fits the data well ($r \approx 0.981$)

Use lots of decimal places here

- c.) Express your regression equation in the form $A(x) = A_0 e^{k \cdot x}$.

We know $A_0 = 0.143$

and $1.92655^x = e^{kx}$

$$\ln(1.92655)^x = \ln e^{(kx)}$$

$$x \cdot \ln(1.92655) = kx \cdot \ln e$$

$$\ln(1.92655) = k$$

$$k \approx 0.65573$$

This value of k is the exponential growth rate. It is the rate at which Tesla's revenue is increasing.

$$A(x) = 0.14 e^{0.65573x}$$

- d.) Use your function to predict Tesla's revenue in 2020.

The year 2020 is 10 years since 2010.

Where x = number of years since 2010

and $A(x)$ = revenue in billions of dollars

$$\rightarrow A(10) = 0.143 e^{0.65573(10)}$$

≈ 100.7 or \$ 100.7 billions

Logarithmic Regression Equations:

expl 2: The data in the table below represent the percent of U.S. citizens aged 12 and over who have a profile on at least one social network. Answer the following questions.

- a.) Use your calculator to draw a scatter plot using x to be the number of years since 2000.

Notice how the numbers in the table are increasing quickly at first and then more slowly.

window
[0, 20] x [0, 100]

Year	Percent on Social Network
2008 ($x = 8$)	10
2009 ($x = 9$)	21
2010 ($x = 10$)	44
2011 ($x = 11$)	53
2012 ($x = 12$)	57
2013 ($x = 13$)	63
2014 ($x = 14$)	66
2015 ($x = 15$)	70
2016 ($x = 16$)	77
2017 ($x = 17$)	80
2018 ($x = 18$)	77
2019 ($x = 19$)	79

- b.) Use your calculator to build a logarithmic regression equation. Graph this function on top of the scatter plot to check it visually. Does it fit the data?

$\text{LnReg} \rightarrow y = -143.219 + 78.392 \cdot \ln x$
 $r \approx 0.959$ so it fits data well.

- c.) Use your function to predict the percent of the U.S. on social media in 2020.

2020 $\rightarrow x = 20$ (20 years since 2000)

$y = -143.219 + 78.392 \cdot \ln 20$

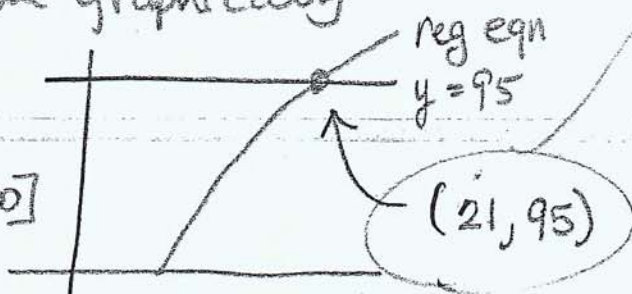
$y \approx 91.62\%$ — used Value func under
2nd TRACE (Calculate menu)

- d.) When does the model predict that 95% of this population will be on social media?

$95 = -143.219 + 78.392 \cdot \ln x$

Solve graphically

window
[0, 25] x
[0, 100]



We should reach 95% on social media in the year 2021. (21 is rounded from 20.88).

Logistic Regression Equations and Carrying Capacity:

expl 3: The data in the table below represent the world's population. Answer the following questions.

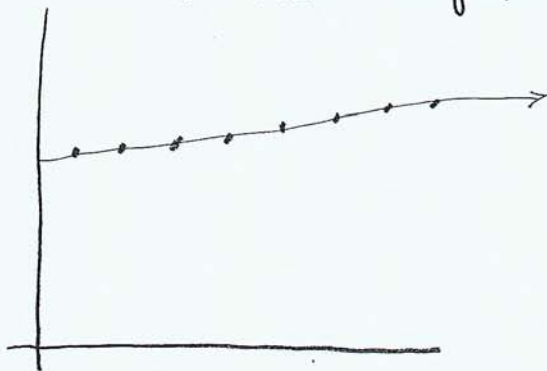
- a.) Use your calculator to draw a scatter plot using x to be the number of years since 2000.

window $[0, 20] \times [0, 10]$

Year	World Population (billions)
2002	6.30
2004	6.46
2006	6.62
2008	6.79
2010	6.96
2012	7.13
2014	7.30
2016	7.47
2018	7.63

- b.) Use your calculator to build a logistic regression equation. Graph this function on top of the scatter plot to check it visually. Does it fit the data?

STAT > CALC > Logistic



$$y \approx \frac{15.412}{1 + 1.513e^{-0.02198x}}$$

The reg. eqn does fit the data well.

- c.) According to your regression equation, what is the world's carrying capacity?

The carrying capacity
for the world's population
is 15.412 billion
(or 15,412,000,000 people).

Remember
that was c in
the equation.