We have studied many types of functions and their graphs. Here is a new class of function.

College algebra
Class notes
Exponential Functions and Their Graphs (section 6.3)

**Definition: Exponential Function:**

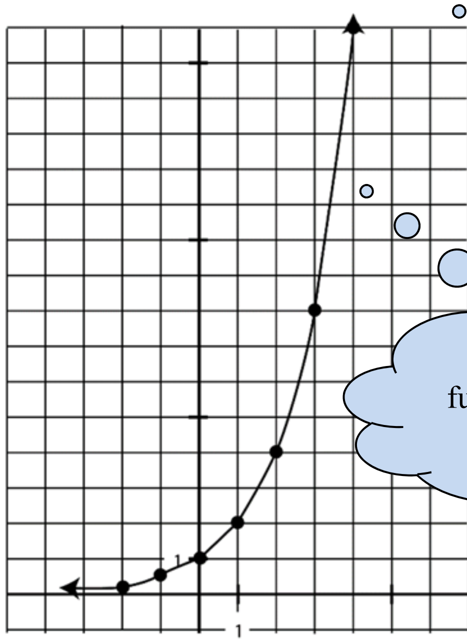
An **exponential function** is of the form $f(x) = a^x$ where $a$ is a positive real number *not* equal to 1. The number $a$ is called the **base** or **growth factor**. Notice the variable $x$ is in the exponent position. This is what makes it an exponential function.

We will also see functions in the form $f(x) = Ca^x$. Here, $C$ is a real number *not* equal to 0 and it is called the **initial value**. Do you see why?

**Domain:** Since you could raise a positive number to any power and get a real number out, there are no real numbers we need to exclude from the domain. So, **the domain of any exponential function is all real numbers**.

expl 1: Consider $y = 2^x$. Inspect the table and graph.

| ***x*** | $y = 2^x$ |
|---|---|
| -2 | $2^{-2} = \frac{1}{4}$ |
| -1 | $2^{-1} = \frac{1}{2}$ |
| 0 | $2^0 = 1$ |
| 1 | $2^1 = 2$ |
| 2 | $2^2 = 4$ |
| 3 | $2^3 = 8$ |
| 4 | $2^4 = 16$ |



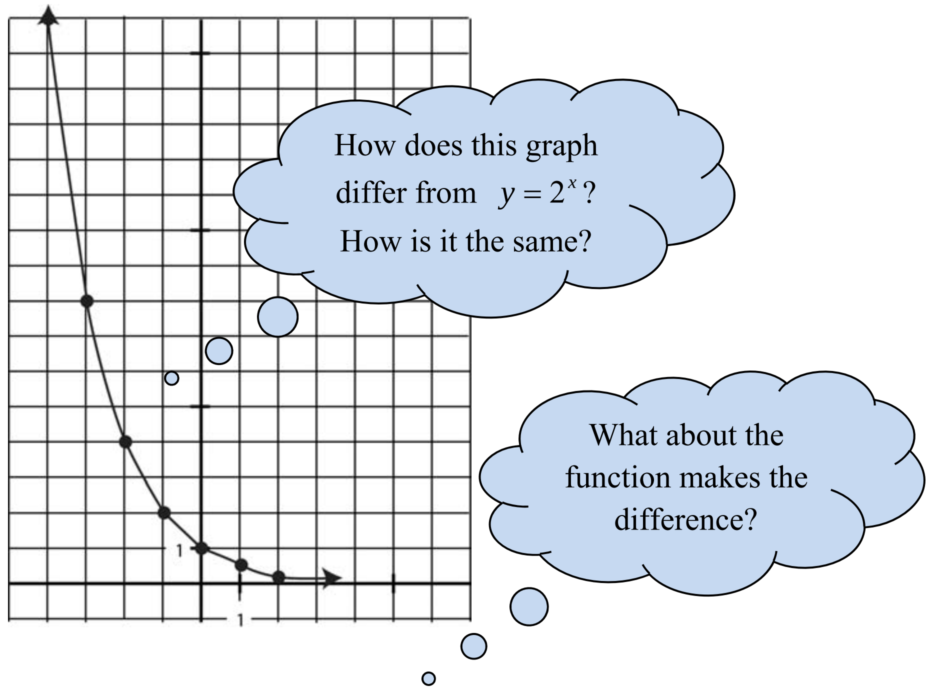
Compare this table and graph to that of a linear function.

All exponential functions will have this general shape.

I call the shape “shloopy”.

expl 2: Consider $y = \left(\frac{1}{2}\right)^x$. Inspect the table and graph.

| **x** | $y = \left(\frac{1}{2}\right)^x$ |
|---|---|
| -4 | $\left(\frac{1}{2}\right)^{-4} = 2^4 = 16$ |
| -3 | $\left(\frac{1}{2}\right)^{-3} = 2^3 = 8$ |
| -2 | $\left(\frac{1}{2}\right)^{-2} = 2^2 = 4$ |
| -1 | $\left(\frac{1}{2}\right)^{-1} = 2^1 = 2$ |
| 0 | $\left(\frac{1}{2}\right)^{0} = 1$ |
| 1 | $\left(\frac{1}{2}\right)^{1} = \frac{1}{2}$ |
| 2 | $\left(\frac{1}{2}\right)^{2} = \frac{1}{4}$ |




Why does $y = 2^x$ increase and $y = \left(\frac{1}{2}\right)^x$ decrease? What do you think makes the difference?

Predict whether the following functions would increase or decrease.

a.) $y = 4^x$

b.) $y = 13^x$

c.) $y = \left(\frac{1}{3}\right)^x$

d.) $y = .67^x$

e.) $y = \left(\frac{5}{4}\right)^x$

**Definition: Natural Exponential Function:**
This is a special exponential function whose base is $e$, an irrational number (meaning its decimal form does *not* terminate or repeat) approximately equal to 2.71828. The **natural exponential function** is $f(x) = e^x$.

expl 3: Find the following using a calculator. Round to four decimal places.

a.) $e^{-5}$

> Calculator: You will see $e^x$ is the second function of the LN button on the left side.

b.) $\left(\dfrac{1}{e^3}\right)^2$

**Optional Worksheet: Working with Exponential Functions**
This worksheet will explore the two shapes we see with the graphs of exponential functions. It will also review simplifying exponents and using function notation. Solutions are available online.

**Review of Exponent Rules:** You will use many of these rules as you manipulate expressions.

Product rule: $a^m \cdot a^n = a^{m+n}$

Quotient rule: $\dfrac{a^m}{a^n} = a^{m-n}$

Power rule: $\left(a^m\right)^n = a^{m \cdot n}$

Power of a product rule: $(a \cdot b)^n = a^n \cdot b^n$

Power of a quotient rule: $\left(\dfrac{a}{c}\right)^n = \dfrac{a^n}{c^n}$

Zero exponent rule: $a^0 = 1$ (Here $a$ cannot be $0$ because $0^0$ is undefined.)

Negative exponent rule: $a^{-n} = \dfrac{1}{a^n}$ and $\dfrac{1}{a^{-n}} = a^n$ (if $a$ is non-zero and $n$ is an integer).

expl 4: Graph this function on the calculator. Use the standard window. This graph can be thought of as a transformation. Can you see the mother function $y = 3^x$ in this graph?

$y = 3 - 3^x$

We can use transformations to help graph these functions by hand. Recall the general shape of $f(x) = a^x$ when $0 < a < 1$ and when $a > 1$. Draw them right now below. What is the $y$-intercept of each graph? Do they have horizontal or vertical asymptotes?

expl 5: Use transformations to sketch the graph of the following functions. Check on your calculator.

a.) $y = 2^x + 1$

b.) $f(x) = -3^{x-2}$

**Worksheet: Exploring Exponential Functions:**
We will look at the graphs of exponential functions, including those involving transformations. We will also practice finding exponential values using the calculator.

expl 6: The value of a stock is given by the function $V(t) = 58\left(1 - e^{-1.1t}\right) + 20$ where $V$ is the value of the stock after time $t$, in months.

a.) Graph the function. Copy it here.

b.) Find $V(1)$, $V(2)$, $V(4)$, $V(6)$, and $V(12)$. Interpret these values.

Can you use the VALUE function under CALCULATE?

**Solving Special Exponential Equations:**

What would you say the solution to $5^x = 5^4$ is? Go with your gut. There is actually a pretty simple property that backs up what your gut probably told you.

**Base-Exponent Property:** For any $a > 0$ and $a \neq 1$, we know that $a^x = a^y$ if and only if $x = y$.

So if $3^x = 3^7$, then what must be true?

Let's look at this in action, but we may have to work to get our equations in this perfect form.

expl 7: Solve the following equations.

a.) $\left(\frac{1}{4}\right)^x = \frac{1}{64}$

b.) $4^x \cdot 2^{x^2} = 16^2$