Function notation and the concept of function will follow you throughout algebra.

College Algebra
Class Notes
Functions (section 3.1)

Idea behind Functions:

Equations like $y = 4x + 5$ or $x^2 + y^2 = 16$ show relationships between variables. These are called **relations**. They can also be represented by a table of values, a list of ordered pairs, or a graph which is just a picture of those ordered pairs. A function is a special kind of relation. Let's review some terminology to help us understand how they are special.

Definition: Domain: the set of all x values (that will give you a real number out for y)

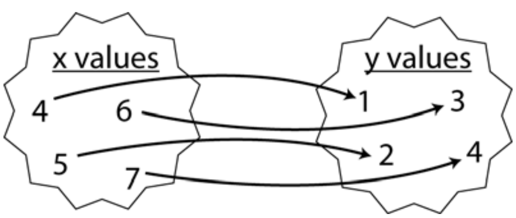
Definition: Range: the set of all y values (that you can get out for y)
Range values are sometimes called **images**.

x-values: inputs
y-values: outputs

But do you remember what a real number is? What would cause the output to be **non-real**?

expl 1: Consider the sets of ordered pairs and their illustrations below. Determine the domains and ranges of these relations.

a.) (4, 1), (5, 2), (6, 3), and (7, 4)



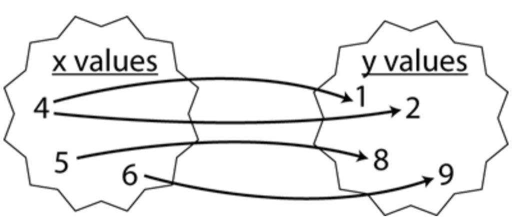
Each x value is assigned to a **specific y value.**

We call this a **mapping**.

What is the domain? What is the range? Write your answers in set notation.

b.) (4, 1), (4, 2), (5, 8), and (6, 9)

The x value of 4 is assigned to **two different y values**.



What is the domain? What is the range? Write your answers in set notation.

Definition: Function: a relation where every x value in the domain is assigned to *exactly one* y *value*. **(**If y is isolated in the equation, we call it **explicitly defined**. If y is *not* isolated, the function is said to be given **implicitly**.)

In example 1 above, which relation is a function and which is *not*? Explain.

Definition: Dependent and Independent Variables: Since we think of most functions in the form of "y = some rule involving x", we think of the x values as **inputs** and the y values as **outputs**. Also, we may say that the value of y *depends* on the value of x. Hence, we will refer to x as the **independent variable** and to y as the **dependent variable**.

expl 2: Which of the following relations are functions?

Complete the table of values.

a.)

x	**y**
1	1
2	2
3	3
4	4

b.) (4, 5)
(5, 6)
(6, 7)
(7, 4)
(4, 7)

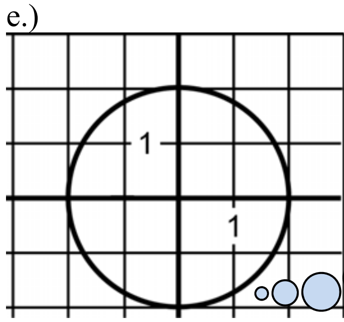
c.) $y = x^2$

x	**y**
-2	
0	
2	
4	

Solve for y.

d.) $x + 2y^2 = 1$

e.)



f.) *domain:* a set of family members
correspondence: their age
range: a set of integer values

Highlight the points for the x value of 1. Does that help?

Interpretation: You can think of a function in a few different ways.

1. a **relationship** between two variables, x and y,
2. a **rule** that tells you what to do to an x value to get out a y value, or
3. a **machine** that produces a y value when you input an x value.

In certain applications, one understanding of function may serve us better than the others.

$y = 2x^2 + 4$

Relationship: Every y value is equal to the x value squared, multiplied by 2, and added to 4.

Rule: This $y = 2x^2 + 4$ tells us what to do to an x value to make a y value. Can you picture a **machine** doing these operations?

Function notation:

Check to see if the following relationships are functions.

Does any x value result in more than one y value?

$y = 2x^2 + 4$		$y^2 = x$ (solved as $y = \pm\sqrt{x}$)	
x	$y = 2x^2 + 4$	x	$y = \pm\sqrt{x}$
-3		9	
0		16	
3		25	
Is y a function of x?		Is y a function of x?	

Since the first relationship is a function, we can use function notation to make sure everyone knows. So we replace the y with $f(x)$ to write $f(x) = 2x^2 + 4$. Sometimes we use different letters like $g(x)$ or $h(x)$, especially if we have more than one function.

We say y is a "function of x".

pronounced "f of x"

expl 3a: Find $f(0)$, $f(-2)$, and $f(5)$ for the function $f(x) = 2x^2 + 4$.

What we want are the y values when x is 0, -2, and 5.

How do you square -2 on the calculator?

Common Mistakes with Notation: As we use function notation in more complicated ways, understanding the notation and using it correctly will be crucial. For instance, in the previous example, we must *never* write $f(x) = 54$ or $f(5) = 2x^2 + 4$. Whatever you write in the parentheses should be substituted for x in the formula at the same time, on the same line.

expl 3b: Recall that the numbers 0, -2, and 5 are x values and the $f(x)$ outputs are their corresponding y values. Write your results from part a in ordered pair notation.

expl 3c: Consider our function $f(x) = 2x^2 + 4$. Find $f(-x)$, $f(x+3)$ and $f(x+h)$.

Apply the rule of f to these numbers.

expl 4: Forensic science uses the function $H(x) = 2.59x + 47.24$ to estimate the height $H(x)$ of a woman (in centimeters) given the length x (in centimeters) of her femur bone.

a.) Estimate the height of a woman whose femur bone measured 40 cm. Round your answer to two decimal places.

What is $H(x)$? What is x? Which are you given and which do you need to find?

b.) I am 5' 5" (or 165.1 centimeters). How long would you expect my femur to be? Round your answer to two decimal places.

Review of Interval Notation:

Do you remember interval notation? Provide each real number line graph and interval notation for these sets of numbers. The real number line graphs help me to visualize the sets.

"the numbers in between 0 and 4, *not* including 0, but including 4"

$<$ is less than
$>$ is greater than

Inequality Notation	Graph on Number Line	Interval Notation
$0 < x \leq 4$		
$x < -4$		
$x \geq 0$		
$x > 5$		
$4 \leq x$		

Sometimes the variable is on the right. How is that different?

smallest number in set **,** largest number in set

square bracket: includes endpoint
parenthesis: does *not* include endpoint

Which is x values?
Which is y?

Finding domain and range:

expl 5: Find the domains of the functions below. Use interval notation.

a.) $y = \dfrac{3}{x+4}$

b.) $h(x) = \sqrt{2x+6}$

c.) $y = 5x + 9$

Instead of asking what x could be, ask yourself what x **cannot** be. The domain is everything else.

Operations on functions:

We'll learn how to add, subtract, multiply, and divide two functions. This next example gives us a good reason.

expl 6: A company knows that the revenue $R(x)$, in dollars, from selling x hundred laptops is $R(x) = -1.2x^2 + 220x$. The cost of making and selling x hundred laptops is $C(x) = 0.05x^3 - 2x^2 + 65x + 500$.

a.) Find the profit $P(x)$ for this company where $P(x) = R(x) - C(x)$.

Revenue is the money they bring in. **Cost** is the money they spend. **Profit** is the difference.

b.) Find and interpret $P(25)$.

Notation: The following notation is often used.

$$(f+g)(x) = f(x) + g(x) = f + g$$
$$(f-g)(x) = f(x) - g(x) = f - g$$
$$(f \cdot g)(x) = f(x) \cdot g(x) = f \cdot g = fg$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{f}{g}, \quad g(x) \neq 0$$

Alternative forms

Domains: all real numbers in both domains of f and g and, in the case of $(f/g)(x)$, exclude those numbers that make $g(x) = 0$.

Why does it matter if $g(x) = 0$?

expl 7: Let $f(x) = 2x^2 + 7x$ and $g(x) = 3x - 5$. Find the following and their domains.

a.) $f + g$

The domain is the set of numbers in *both* domains of f and g.

For the quotient, we *exclude* those x's that would cause division by zero.

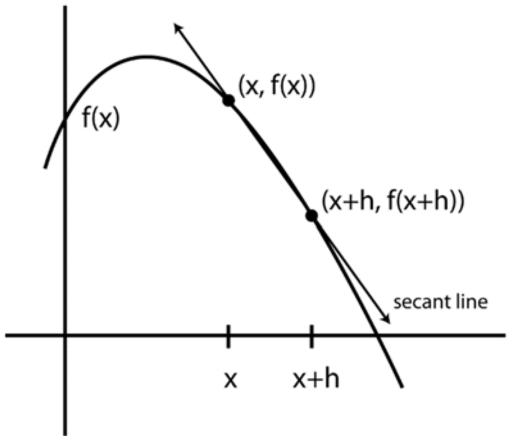
b.) f / g

expl 8: Let $f(x) = \sqrt{x}$ and $g(x) = 3x - 5$. Find the following if they exist.

a.) $(f \cdot g)(9)$

b.) $(f / g)(5/3)$

Difference Quotient: For a function $f(x)$, we can define two points on the graph shown below. We find the **slope of the secant line** through the points and we end up at the difference quotient. (Notice that h would be assumed to be non-zero. Do you see why?)



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{f(x+h) - f(x)}{x + h - x}$$

$$= \frac{f(x+h) - f(x)}{h}$$

It's used in calculus to approximate how fast a function is changing.

It's nice to know where it comes from, but in practice you just need to know how to use it. Use the circled formula above when asked to find the difference quotient for a function.

expl 9: Find the difference quotient for the following function.
$f(x) = 3x^2 - 5$

What is $f(x + h)$? Now complete the formula.

How do we subtract $f(x)$ from $f(x + h)$?