

A function turns an  $x$  into a  $y$  value. How do we reverse that?

**Main Idea:** Consider the function given below. What does it do to each  $x$  value to get its corresponding  $y$  value? How would we reverse the process? In other words, what would we do to a  $y$  value to get its  $x$  value back? That is what a function's inverse does for us.

Consider the function  $y = 3x + 4$ . Below is a verbal model that shows the relationship between  $x$  and  $y$ .



The following table shows some  $x$  values and their corresponding  $y$  values.

$x$	$y = 3x + 4$
-2	$y = 3(-2) + 4 = -2$
-1	$y = 3(-1) + 4 = 1$
0	$y = 3(0) + 4 = 4$
1	$y = 3(1) + 4 = 7$
2	$y = 3(2) + 4 = 10$

Each  $y$  value is gotten by multiplying the  $x$  value by 3 and adding 4.

But what if we wanted to go the other way? What would you do to the ( $y$  value of) 10 to get back to the ( $x$  value of) 2?

We want to reverse this process. If the original process multiplies by 3 and adds 4, how would you reverse that? Does the order of your operations matter?

Write down a formula for your inverse.

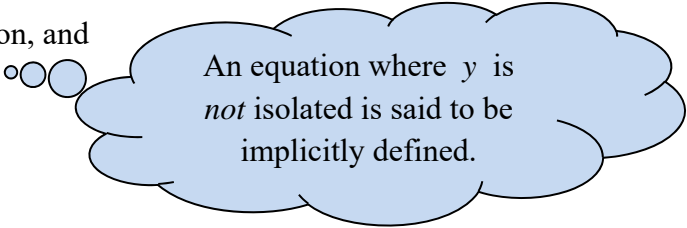
An inverse **undoes** what the original function did.

**Variables of Inverses:** Let's think about our use of  $x$  and  $y$ . We denote inputs as  $x$  and outputs as  $y$ . This is true for the original  $y = 3x + 4$ . However, when we think about its inverse, we essentially switch the roles of  $x$  and  $y$ , using  $y$  values as inputs and  $x$  values as outputs. This idea with the previous discussion leads to a general scheme for algebraically finding inverses.

**Algebraic Steps for Finding Inverses:**

To find the inverse of a function,

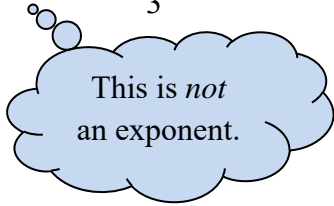
1. Switch the  $x$  and  $y$  in the equation, and
2. Solve the equation for  $y$ .



An equation where  $y$  is *not* isolated is said to be implicitly defined.

Try this procedure with  $y = 3x + 4$ .

**Notation:** If we let  $f(x)$  be a function, then we can call its inverse  $f^{-1}(x)$ . This is pronounced simply “ $f$  inverse of  $x$ ”. For instance, we could write  $f(x) = 3x + 4$  and  $f^{-1}(x) = \frac{x-4}{3}$ .



This is *not* an exponent.

expl 1: Complete the following to investigate inverses.

a.) Use the function  $f(x) = 3x + 4$  to find  $f(12)$ .

b.) Use the function  $f^{-1}(x) = \frac{x-4}{3}$  to find  $f^{-1}(40)$ .

c.) Explain the connection between  $f(12)$  and  $f^{-1}(40)$ .

expl 2: Find the inverse of each relation.

a.)  $\{ (3, 2), (-5, 4), (1, 6) \}$

b.)  $x^3 y = 15$

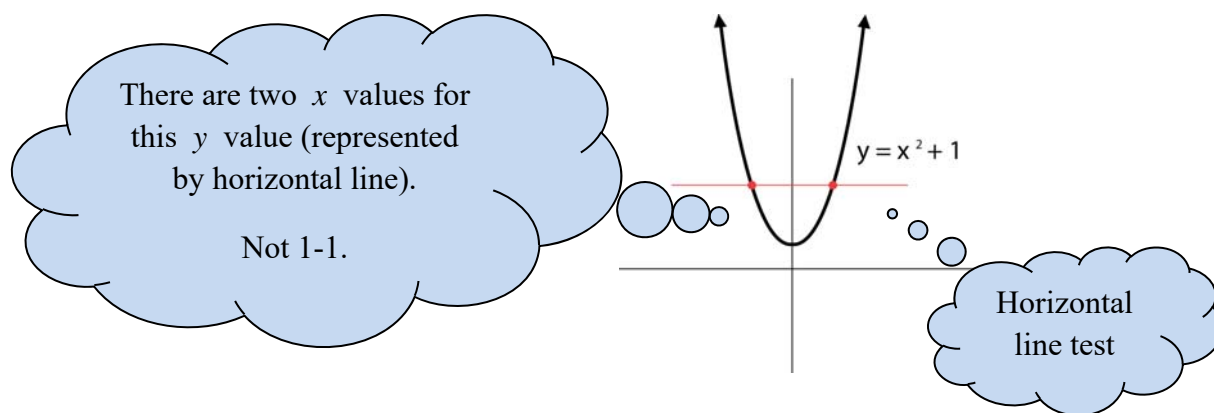
c.)

State	Unemployment Rate
Virginia	2.9%
Nevada	4.0%
Tennessee	3.2%
Texas	3.7%

Source: US Bureau of Labor Statistics, April 2019

### Definition: One-to-One Functions:

A function is **one-to-one (1-1)** if for every  $y$  value, there is exactly one  $x$  value. Consider the function below.



**A function is 1-1 if different inputs have different outputs.** Or rather, if  $x_1 \neq x_2$ , then  $f(x_1) \neq f(x_2)$ . Another way to state this is, if  $f(x_1) = f(x_2)$ , then  $x_1 = x_2$ . You would use this to prove a function is 1-1 or to algebraically show it is *not*.

expl 3: Graph the function and determine if it is one-to-one. Copy your graphs here.

a.)  $y = |x + 4|$

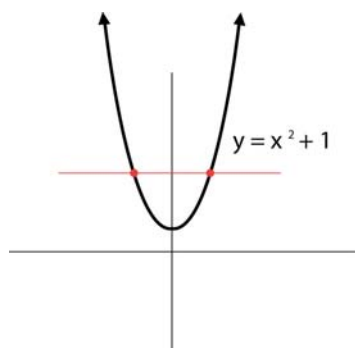
b.)  $g(x) = -x^3 + 2$

### Why is being one-to-one important?

Remember how  $y = x^2 + 1$  was found to *not* be one-to-one? Find the inverse of  $y = x^2 + 1$ . Is this inverse a function?

**Restricted Domains:** It turns out that if a function is *not* 1-1, then its inverse will *not* be a function. And that is a problem because algebra loves functions. What's a person to do?

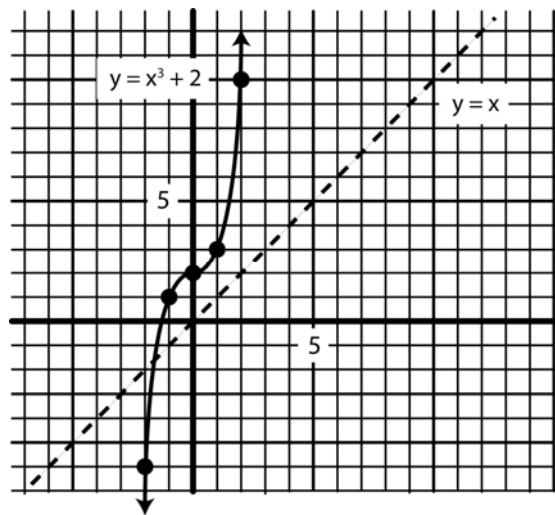
We could restrict the domain of the original function so that its inverse is a function. Consider the function below whose domain is currently "all real numbers". Lop off part of it so that the remaining part is one-to-one. What is the new (restricted) domain?



What is the inverse of this new function, with its restricted domain?

## Graphical Interpretation of Inverses:

expl 4: Find the inverse for the function  $y = x^3 + 2$  pictured below by reversing the ordered pairs.



Reversing the points' coordinates is akin to switching  $x$  and  $y$  to find the inverse.

How are the graphs of a function and its inverse related?

expl 5: Consider the function  $g(x) = \sqrt{x+3}$ . Answer the following questions.

a.) Algebraically find the inverse of  $g$ .

Consider the graphs to check your inverse.

b.) Graph both functions on the same plane. Determine the domain and range for both  $g$  and  $g^{-1}$ .

How are the domains and ranges related?

**Graphing functions with restricted domains:** Your calculator will graph with restricted domains. For instance, let's graph the function and its inverse from the previous example.

Enter the following into the calculator.

$$Y_1 = \sqrt{x+3}$$

$$Y_2 = (x^2 - 3)(x \geq 0)$$

We would write this  
as  $y = x^2 - 3, x \geq 0$   
on paper.

The inequality symbol is found  
under TEST which is the  
second function of MATH.

Use the Zoom:  
ZSquare setting.

expl 6: For the function  $f$ , use composition to show that  $g$  is its inverse. In other words, show that  $f(g(x)) = x$  and  $g(f(x)) = x$ .

$$f(x) = \frac{2}{5}x + 1$$

$$g(x) = \frac{5x - 5}{2}$$

Can you see how this  
shows each function  
undoes the other?

You may need  
to review  
composition.

Do we need to exclude  
values from the  
domains of  $f$  or  $g$ ?