When might we want to add two functions? Or multiply them? The difference quotient is used in calculus.

College algebra
Operations on Functions and the Difference Quotient
Section 2.2

Operations on functions: We'll learn how to add, subtract, multiply, and divide two functions. This first example gives us a good reason.

expl 1: A brick company has two factories. The first factory has a total cost of $C_1(x) = 3x + 40$ where x is the number of units made. The second factory has a total cost of $C_2(x) = 4x + 50$.

a.) What is the total cost of both factories?

What operations do the questions imply?

b.) How much more does factory 2 cost to operate?

Notation: The following notation is often used.

$$(f+g)(x) = f(x) + g(x) = f + g$$
$$(f-g)(x) = f(x) - g(x) = f - g$$
$$(f * g)(x) = f(x) * g(x) = f * g = fg$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{f}{g}, \quad g(x) \neq 0$$

Alternative forms

Domains: all real numbers in both domains of f and g and, in the case of $(f/g)(x)$, exclude those numbers that make $g(x) = 0$

Why does it matter if $g(x) = 0$?

expl 2: Let $f(x) = \sqrt{x}$ and $g(x) = 3x - 5$. Find the following and their domains.

a.) $f + g$

> The domain is the set of numbers in both domains of f and g.
>
> For the quotient, we exclude those x's that would cause division by zero.

b.) $f - g$

c.) f / g

expl 3: Let $f(x) = \sqrt{x}$ and $g(x) = 3x - 5$. Find the following if they exist.

a.) $(f\,g)(9)$

b.) $(f / g)(4)$

c.) $f(9) + g(9)$

expl 4: Use the graphs to the right.

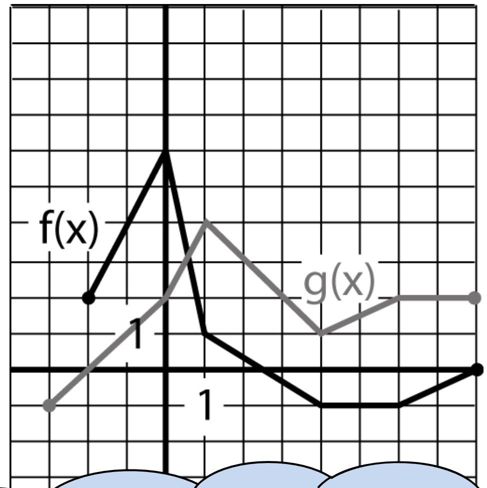
a.) Find the domain of g.

b.) Find the domain of f.

c.) Find the domain of $f+g$.

d.) Graph $f+g$.

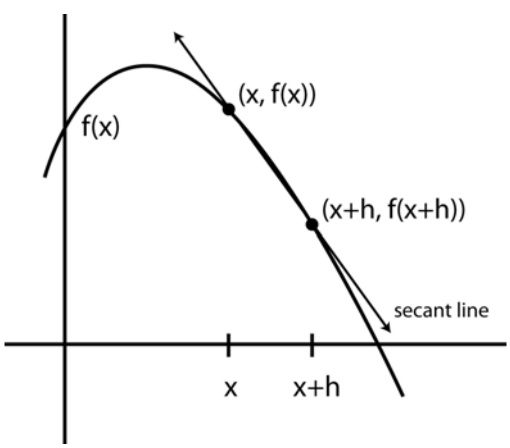
e.) Find the domain of f/g.



For which x values could you find $f(x)+g(x)$? For instance, can you find $f(-3)+g(-3)$? Can you find $f(4)+g(4)$?

How does the domain of f/g differ from that of $f+g$?

Difference Quotient: For a function $f(x)$, we can define two points on the graph shown below. We find the **slope of the secant line** through the points and we end up at the difference quotient.



$$m=\frac{y_2-y_1}{x_2-x_1}$$
$$=\frac{f(x+h)-f(x)}{x+h-x}$$
$$=\frac{f(x+h)-f(x)}{h}$$

It's used in calculus to approximate how fast a function is changing.

It's nice to know where it comes from, but in practice you just need to know how to use it. Use the circled formula above when asked to find the difference quotient for a function.

expl 5: Find the difference quotient for the following function.

$f(x) = 3x^2 - 5x$

What is $f(x+h)$? Now complete the formula.

How do we subtract $f(x)$ from $f(x+h)$?