When a function turns an x into a y value, how do we reverse the process?

College Algebra
Class Notes
Inverses and One-to-One Functions (section 5.1)

Main Idea: Consider the function given below. What does it do to each x value to get its corresponding y value? How would we reverse the process? In other words, what would we do to a y value to get its x value back? That is what a function's inverse does for us.

Consider the function $y = 3x + 4$. Below is a verbal model that shows the relationship between x and y.

Start x → Multiply by 3 → Add 4 → End y

The following table shows some x values and their corresponding y values.

x	$y = 3x + 4$
-2	$y = 3(-2) + 4 = -2$
-1	$y = 3(-1) + 4 = 1$
0	$y = 3(0) + 4 = 4$
1	$y = 3(1) + 4 = 7$
2	$y = 3(2) + 4 = 10$

Each y value is gotten by multiplying the x value by 3 and adding 4.

But what if we wanted to go the other way? What would you do to the (y value of) 10 to get back to (the x value of) 2?

We want to reverse this process. If the original process multiplies by 3 and adds 4, how would you reverse that? Does the order of your operations matter?

Write down a formula for your inverse.

An inverse **undoes** what the original function did.

Variables of Inverses: Let's think about our use of x and y. We denote inputs as x and outputs as y. This is true for the original $y = 3x + 4$. However, when we think about its inverse, we essentially switch the roles of x and y, using y values as inputs and x values as outputs. This idea with the previous discussion leads to a general scheme for algebraically finding inverses.

Algebraic Steps for Finding Inverses:
To find the inverse of a function,

1. Switch the x and y in the equation, and
2. Solve the equation for y.

Some problems will not require you isolate y.

Try this procedure with $y = 3x + 4$.

Notation: If we let $f(x)$ be a function, then we can call its inverse $f^{-1}(x)$. This is pronounced simply "f inverse of x". For instance, we could write $f(x) = 3x + 4$ and $f^{-1}(x) = \frac{x-4}{3}$.

This is **not** an exponent.

expl 1: Complete the following to investigate inverses.
a.) Use the function $f(x) = 3x + 4$ to find $f(12)$.

b.) Use the function $f^{-1}(x) = \frac{x-4}{3}$ to find $f^{-1}(40)$.

c.) Explain the connection between $f(12)$ and $f^{-1}(40)$.

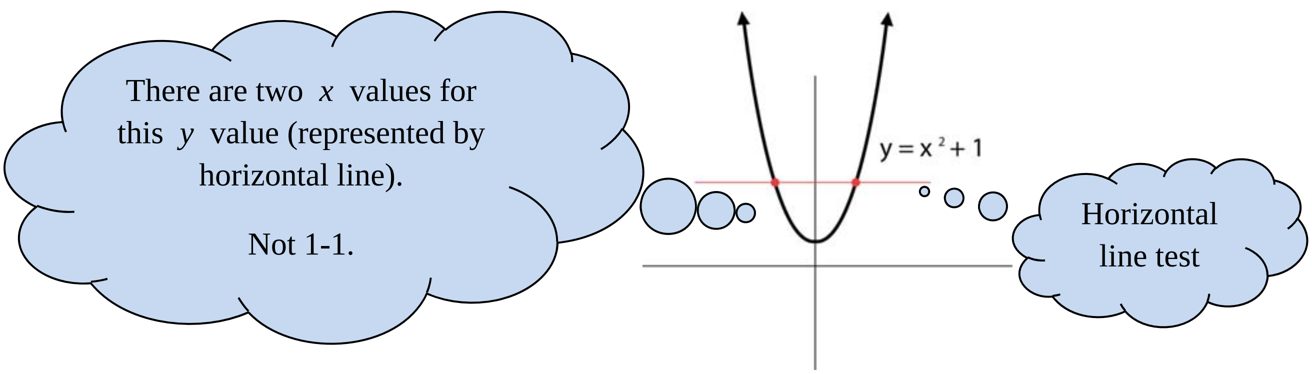
expl 2: Find the inverse of the relation. You do not need to solve part *c* for y.

a.) { (3, 2), (-5, 4), (1, 6) }

c.) $x^3 y = 15$

b.) $y = 4x - 5$

Definition: One-to-One Functions:
A function is **one-to-one (1-1)** if for every y value, there is exactly one x value. Consider the function below.



A function is 1-1 if different inputs have different outputs. Or rather, if $a \neq b$, then $f(a) \neq f(b)$. Another way to state this is, if $f(a) = f(b)$, then $a = b$. We'll use this to prove a function is 1-1 or to algebraically show it is not.

expl 3: Graph the function and determine if it is one-to-one. Copy your graphs here.

a.) $y = |x+4|$

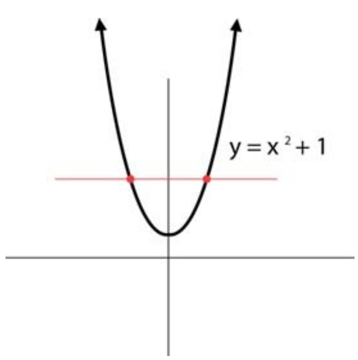
b.) $g(x) = -x^3 + 2$

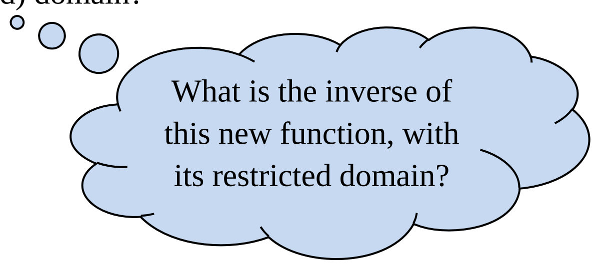
Why is being one-to-one important?

Remember how $y = x^2 + 1$ was found to **not** be one-to-one? Find the inverse of $y = x^2 + 1$. Is this inverse a function?

Restricted Domains: It turns out that if a function is not 1-1, then its inverse will not be a function. And that is a problem because algebra loves functions. What's a person to do?

We could restrict the domain of the original function so that its inverse is a function. Consider the function below whose domain is currently "all real numbers". Lop off part of it so that the remaining part is one-to-one. What is the new (restricted) domain?



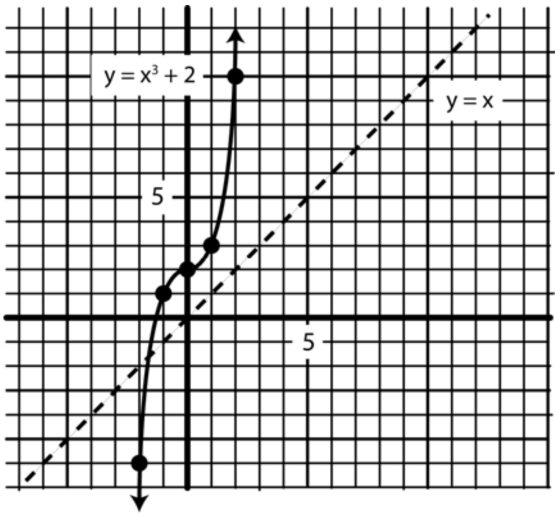


Worksheet: Inverses of functions 2
This worksheet will provide practice for finding an inverse and exploring the concept of one-to-one functions and restricted domains.

Optional Worksheet: Inverses of functions
This worksheet discusses finding inverses graphically and algebraically. It also discusses one-to-one functions and restricted domains.

Graphical Interpretation of Inverses:

expl 4: Find the inverse for the function $y = x^3 + 2$ pictured below by reversing the ordered pairs.



Reversing the points' coordinates is akin to switching x and y to find the inverse.

How are the graphs of a function and its inverse related?

expl 5: Consider the function $g(x) = \sqrt{x+3}$. Answer the following questions.

a.) Algebraically find the inverse of g.

Consider the graphs to check your inverse.

b.) Graph both functions on the same plane. Determine the domain and range for both g and g^{-1}.

How are the domains and ranges related?

Graphing functions with restricted domains: Your calculator will graph with restricted domains. For instance, let's graph the function and its inverse from the previous example.

Enter the following into the calculator.

$Y_1 = \sqrt{}(x+3)$

$Y_2 = (x^2 - 3)(x \geq 0)$

We would write this as $y = x^2 - 3,\ x \geq 0$ on paper.

The inequality symbol is found under TEST which is the second function of MATH.

Use the Zoom: ZSquare setting.

expl 6: The formula $T(x) = \dfrac{5}{9}(x - 32)$ converts Fahrenheit temperatures x to Celsius temperatures $T(x)$.

a.) Find T(95).

b.) Define x and y for the function $T(x)$.

c.) Find $T^{-1}(x)$ and explain what it represents. In other words, define x and y for this function.

d.) Without doing the calculation, what do you think $T^{-1}(35)$ is equal to? Explain.

These two problems are optional.

expl 7: For the function f, use composition to show that g is its inverse. In other words, show that $f(g(x)) = x$ and $g(f(x)) = x$.

Can you see how this shows each function undoes the other?

You may need to review composition.

$$f(x) = \frac{2}{5}x + 1$$

$$g(x) = \frac{5x - 5}{2}$$

expl 8: Prove the following function is one-to-one.
$f(x) = 4 - 2x$

Let's say there are two numbers a and b in the domain such that $f(a) = f(b)$.

Can you prove that $a = b$?