Meet the straight line and its equation.

College algebra
Linear Functions 1A: Definition, Horizontal and Vertical Lines, Slope, Rate of Change, Slope-intercept Form (section 1.3)

Definition: Linear Equation: a linear equation in two variables is an equation that *could be written* in the form $Ax + By = C$ where A, B, and C are real numbers and A and B are not both zero.

Graphs of linear equations will be perfectly straight lines.

Why would we say that A and B are not both zero?

Which of the following are linear equations?

$3x + 4y = 15$ $\quad$ $y = 4$

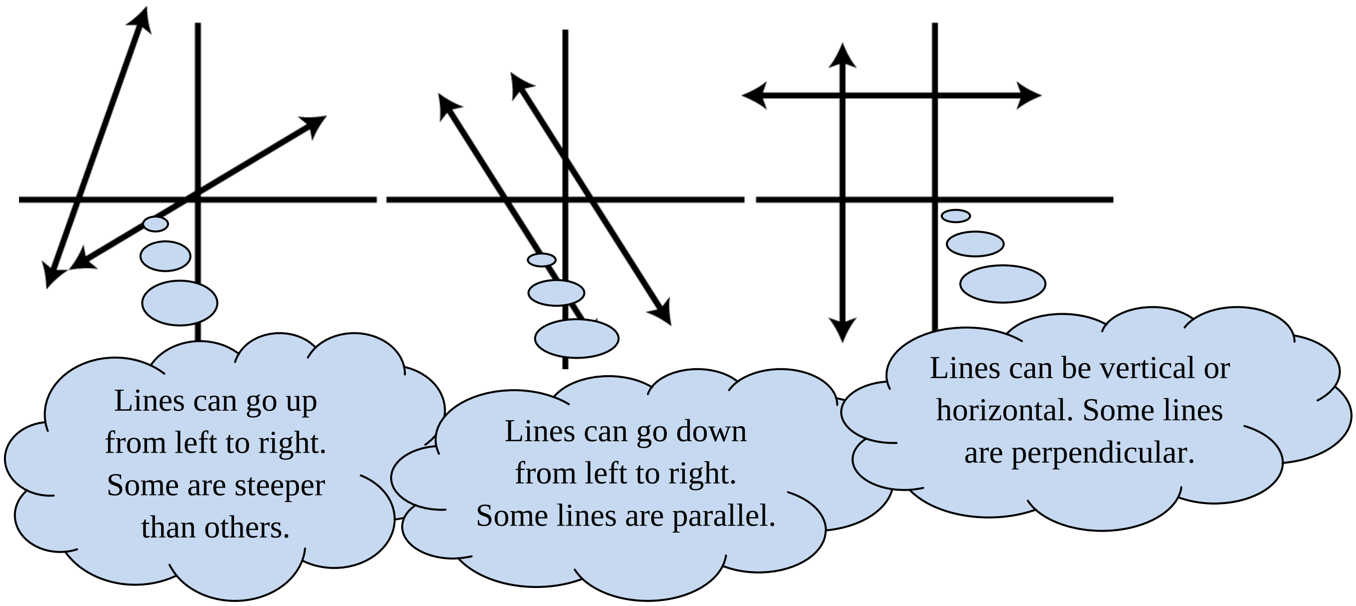
$\frac{1}{2}x - 7y = 10$ $\quad$ $y = 3x + 4$ $\quad$ $6x = 12$ $\quad$ $4x^2 + 5y = 12$

You might try to write the linear equations above in the form $Ax + By = C$, which is called the **standard form**. We will investigate the slope-intercept and point-slope forms in this section.

We will investigate graphs of linear equations here. The idea behind a graph is that it shows every single point that makes the equation true. Another way to say this is that the points "satisfy the equation".

Points are in the form (x, y).

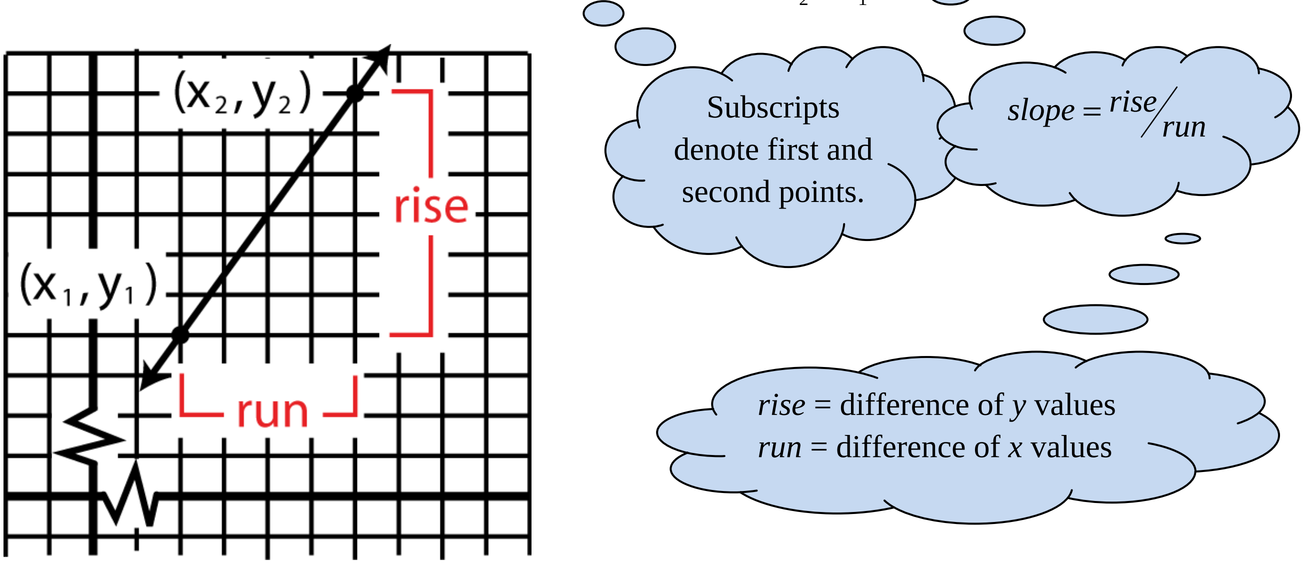
Slope: The slope of a line tells you how slanted it is. Imagine walking up (or down) a line from left to right and you understand why that is important.



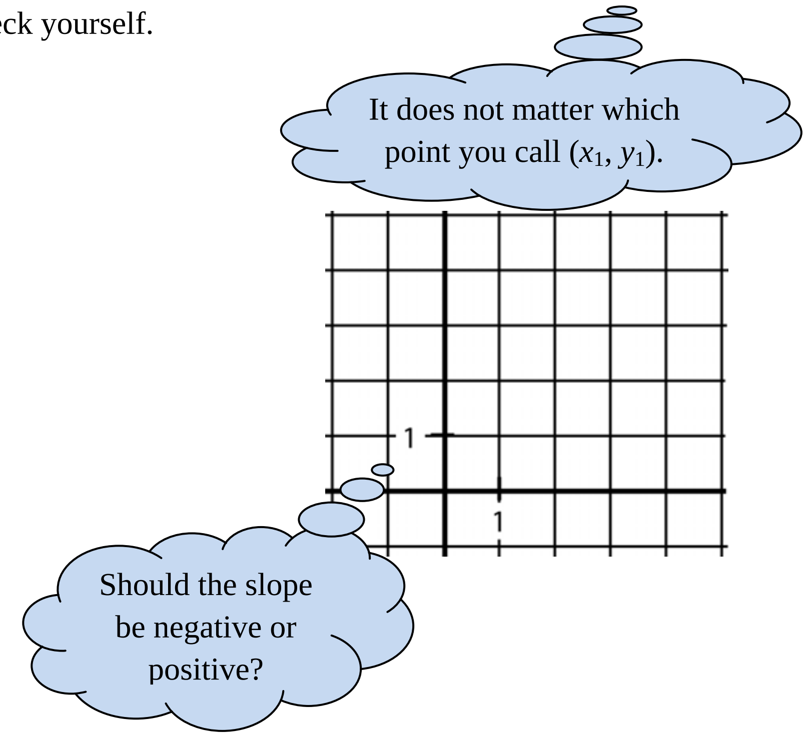
Imagine any two points on a line. **Slope is the ratio of how far we go up (or down) to how far we go right (or left) to get from one point to the other point.** As the steepness of the line changes, this ratio would change too.

Formula for Slope:

The slope between the two points (x_1, y_1) and (x_2, y_2) is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

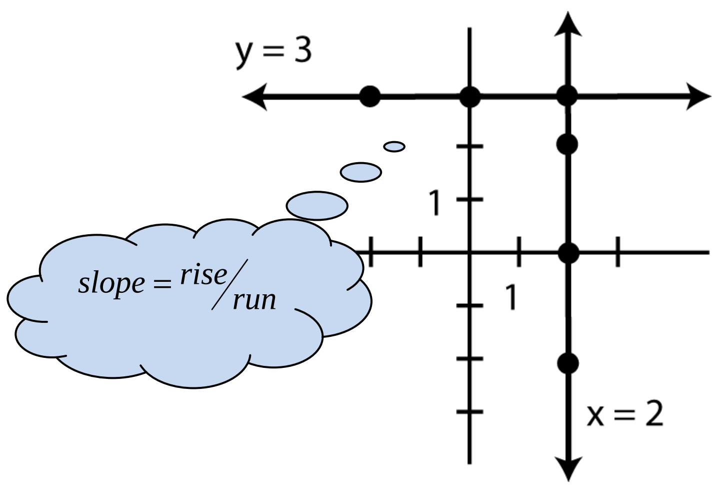


expl 1: Use the formula to find the slope of the line that goes through the points (–1, 4) and (3, 2). Plot the points on the graph to check yourself.



Horizontal and Vertical Lines:

Find the slope of these lines. Rather than using the formula from the previous page, use the quicker "rise over run" method.



Use what you found above to generalize about the slope of all vertical and horizontal lines.

Slope of any vertical line =

Slope of any horizontal line =

Slope-intercept Form of a Line:

Any (non-vertical) line could be written in the form $y = mx + b$. Here, m is the slope and b is the y-intercept. It also helps to think of (x, y) as a generic point on the line.

expl 2: Find the slope and y-intercept of the line $y = -9.3x + 4.7$.

expl 3: Find the slope of the line $x = 5$.

expl 4: Find the slope of the line $y = -8$.

expl 5: Find the slope and y-intercept of the line $2x - 3y = 12$.

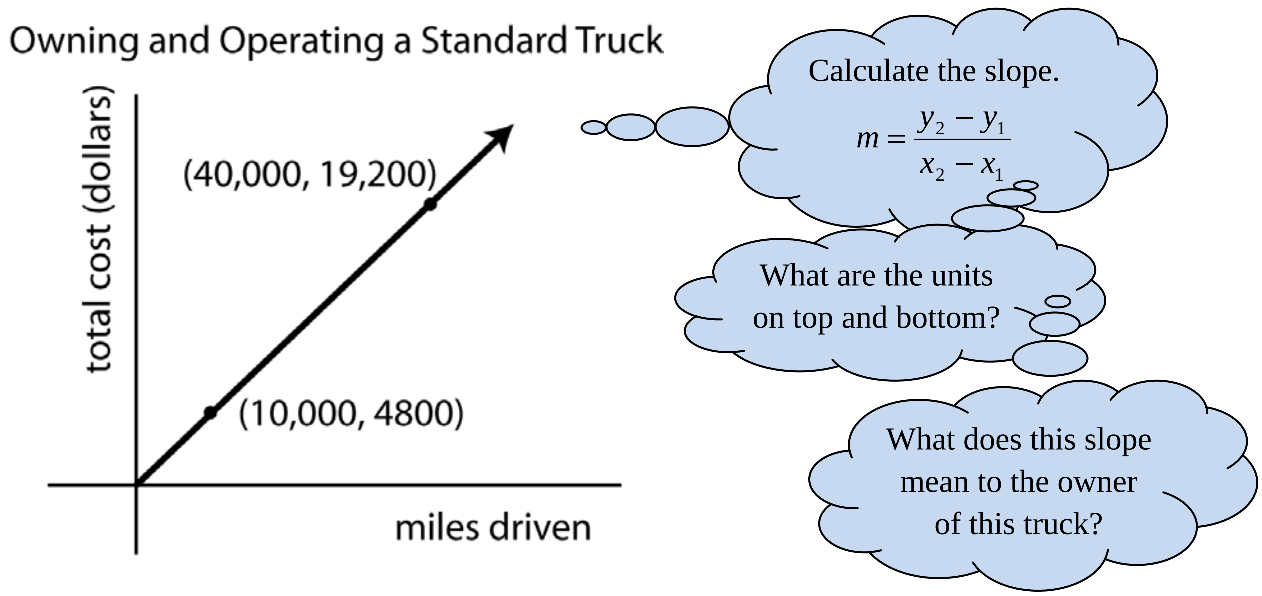
Method 1: Find 2 points on the line and use the formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

Method 2: Rewrite the equation in $y = mx + b$ form and pick out m.

Average Rate of change:

The slope of a line is the difference of the y-values divided by the difference of the x-values. This ratio tells us how fast y is changing with respect to x, or average rate of change.

expl 6: Find the slope of the line and write it as a average rate of change. Don't forget the units.



expl 7: Margie has a pastry shop. For a certain kind of cupcake she produces, she has a variable cost of $1.25 per cupcake and fixed costs of $200. Let x be the number of cupcakes she makes, and let $C(x)$ be the total cost for these cupcakes.

a.) Find an equation for $C(x)$, her total cost.

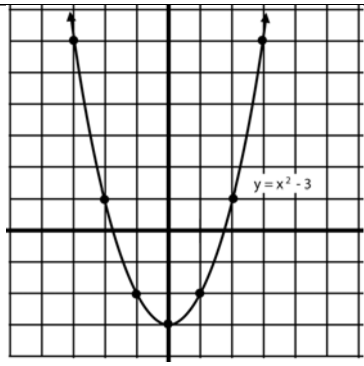
b.) How much will it cost Margie to produce 100 cupcakes?

Straight Line Graphs:

So what makes the linear function graph as a straight line and other functions, like quadratic functions, not? Consider the functions below.

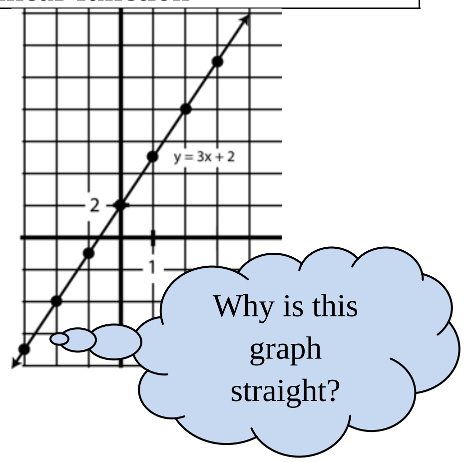
Quadratic function

x	$y = x^2 - 3$
-3	6
-2	1
-1	-2
0	-3
1	-2
2	1
3	6



Linear function

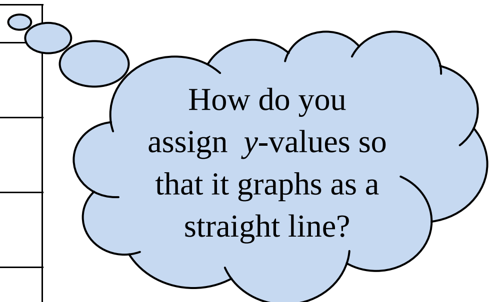
x	$y = 3x + 2$
-3	-7
-2	-4
-1	-1
0	2
1	5
2	8
3	11



What about the y-values in the linear function's table will force the graph to be a straight line?

expl 8: Can you make up a table of values that would be graphed as a straight line? Complete the table below. Make a quick graph too.

x	y
0	
1	
2	
3	
4	
5	



expl 9: Consider a linear function we will call $f(x)$. Answer the following questions.

a.) Find the slope between the two points $f(2) = 5$ and $f(5) = -7$.

Function notation means $f(x) = y$. So which two points are we given? Write them in ordered pair notation.

b.) When we graph these two points and the line between them, we notice that the line hits the y-axis at 13. So what is the (slope-intercept) equation of this line? You may write it as "$y = \ldots$" or "$f(x) = \ldots$".