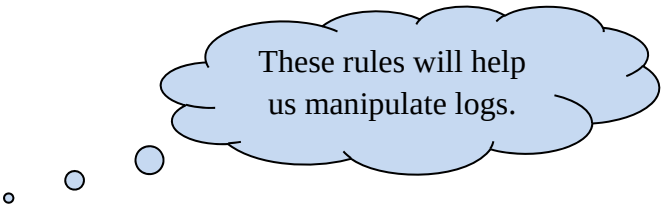
College algebra
Class notes
Logarithm Rules (section 5.4)



Optional Worksheet: Logarithm Rules Worksheet
This worksheet will show you how to derive the formulas given in this section. If we can think through the formulas, they will be easier to memorize and apply. Try to use what you know about logs to figure out the following excerpt from the worksheet.

1. In words, what is $\log_b b$? It's the number to which I raise _____ to get _____.
What does this number we call $\log_b b$ have to be?

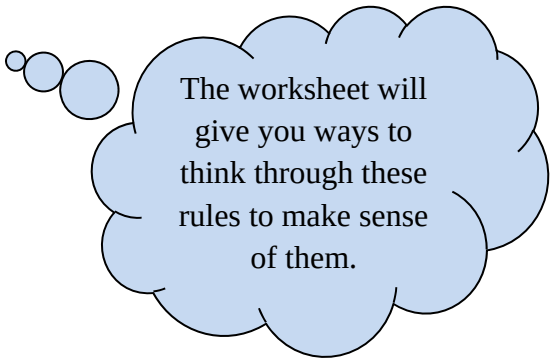
2. In words, what is $\log_b 1$? It's the number to which I raise _____ to get _____.
What does this number we call $\log_b 1$ have to be?

3. In words, what is $\log_b b^k$? It's the number to which I raise _____ to get _____.
What does this number we call $\log_b b^k$ have to be?

4. Now $\log_b v$ is the number to which I raise b to get v. So when we raise b to this number, what should I get? In other words, what is $b^{\log_b v}$?

The complete list of rules is listed below including the Change of Base Formula from the previous section.

1. $\log_a\left(M/N\right) = \log_a M - \log_a N$
2. $\log_a\left(M \cdot N\right) = \log_a M + \log_a N$
3. $\log_a M^p = p \cdot \log_a M$
4. $\log_a a = 1$
5. $\log_a 1 = 0$
6. $\log_a a^x = x$
7. $a^{\log_a x} = x$
8. $\log_b M = \dfrac{\log_a M}{\log_a b}$



Common Mistakes:

It is common to incorrectly assume other rules similar to those given. Be careful when you apply the rules. You should also try out numbers in any rule you "think" is right.

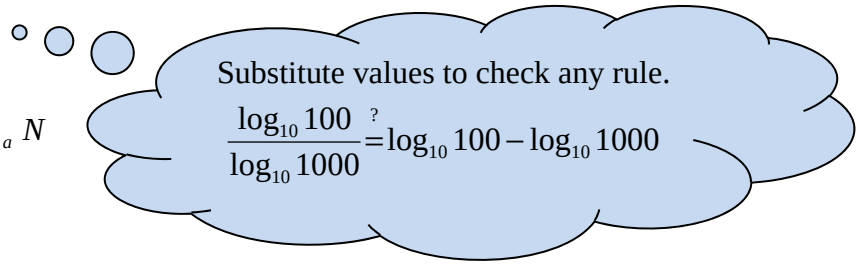
$\dfrac{\log_a M}{\log_a N} \neq \log_a M - \log_a N$

$(\log_a M) * (\log_a N) \neq \log_a M + \log_a N$

$\log_a MN \neq (\log_a M)(\log_a N)$

$(\log_a M)^p \neq p(\log_a M)$

etc...



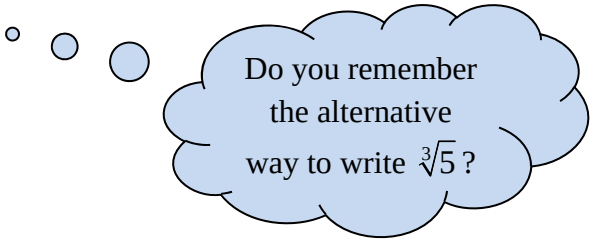
expl 1: Express as a sum of logs. Simplify if possible.

$\log_4(64 \cdot 4)$

expl 2: Express as a product. Simplify if possible.

a.) $\ln y^5$

b.) $\log_3 \sqrt[3]{5}$



expl 3: Express as a difference of logs. Simplify if possible.

$\log \frac{t}{w}$

expl 4: Express as a sum or difference of logs. Simplify if possible.

a.) $\log_b \frac{x^2 y}{b^3}$

Use $\log_a M^p = p \cdot \log_a M$ where applicable.

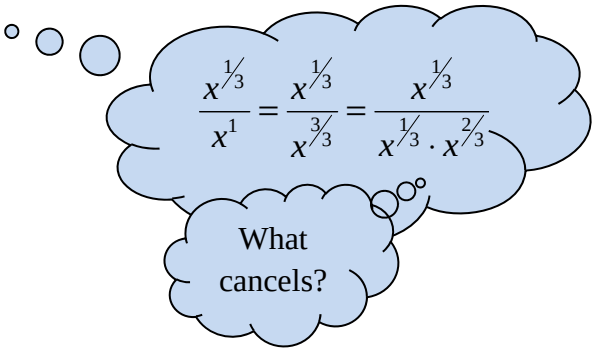
b.) $\log_x \sqrt[4]{\frac{x^8 y^3}{z^{12}}}$

You may want to first simplify $\sqrt[4]{\frac{x^8 y^3}{z^{12}}}$.

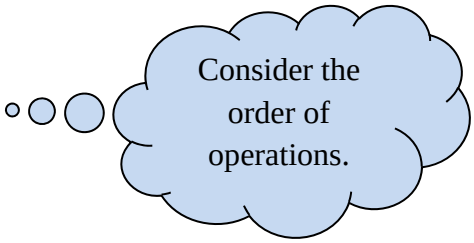
But if you do, recall the rules $\left(\frac{a}{c}\right)^n = \frac{a^n}{c^n}$, $(a \cdot b)^n = a^n \cdot b^n$, and $(a^n)^m = a^{n \cdot m}$.

expl 5: Express as a single log. Simplify if possible.

a.) $\frac{1}{3}\log_a x + 4 \cdot \log_a y - \log_a x$



b.) $\ln x - 3[\ln(x-5) + \ln(x+5)]$



expl 6: Given that $\log_a 2 \approx 0.693$ and $\log_a 7 \approx 0.845$, find the following.

a.) $\log_a 14$

b.) $\log_a 49$

expl 7: Simplify.

a.) $t^{\log_t 3}$

b.) $\ln e^6$

c.) $\log 10^{-k}$

expl 8: Solve the following equations.

a.) $5^{\log_5 8} = 2x$

b.) $\ln e^{3x-5} = -8$

expl 9: It is easy to remember these rules incorrectly. We may need to check our memory. Substitute values for the variables to verify that the following "rules" are true. Use a base of 10 or e so you can use your calculator to evaluate the logs.

a.) $\log_a (M \cdot N) \overset{?}{=} \log_a M + \log_a N$

b.) $\log_a 1 \overset{?}{=} 0$

Worksheet: Visiting with exponential and logarithmic functions:

This worksheet will explore the relationship between exponential functions and their inverses, logarithmic functions. We will also work on understanding what a logarithm means and how that helps us with the logarithm rules.