We will use our log rules and the equivalency of $x = a^y$ and $y = \log_a x$.

College algebra
Class notes
Solving Exponential and Logarithmic Equations (section 5.5)

Definition: Exponential Equation: An equation with the variables in the exponent position will be called an exponential equation.

Examples:

$5^x = 5^4,\qquad e^{2t} = 500,\qquad 3^{4x} = 81,\qquad 3^r = 2^{r-1},\qquad e^x - 6e^{-x} = 1,\qquad 27 = 3^{5x} \cdot 9^{x^2}$

Definition: Logarithmic Equation: An equation that has logs of variable expressions will be called a logarithmic (or log) equation.

Examples:

$\log_3(4w) = 4,\qquad 3\log_2(x-1) + \log_2 4 = 5,\qquad \log_4 x + \log_4(x-3) = 1,\qquad \log_2(x-1) - \log_6(x+2) = 2$

We're solving equations in these sections. Remember we are trying to find the x that makes the equation true. We will need to keep a lot in our minds.

1.) the logarithm rules from the previous section
2.) the fact that $x = a^y$ and $y = \log_a x$ are equivalent
3.) the two properties below

Base-Exponent Property: For any $a > 0$ and $a \neq 1$, we know that $a^x = a^y \longleftrightarrow x = y$.

So if $5^x = 5^4$, then what must be true?

Logarithmic Equality Property: For any $M > 0$, $N > 0$, $a > 0$, and $a \neq 1$, we know that $\log_a M = \log_a N \longleftrightarrow M = N$.

Notice the arrows go both ways. We will make use of that.

So if $\log_2 x = \log_2 10$, then what must be true?

Many of the problems can be solved by various methods. We will explore this with the examples. When doing homework, choose the method that most appeals to you or that best fits that equation.

Solving Exponential Equations:

expl 1: Solve. Try the different methods below.

$4^{2t+1} = 20$

Method 1: Use the equivalency of $x = a^y$ and $y = \log_a x$ to rewrite the equation in log form.

You're not done until t is isolated.

Method 2: Use the **Logarithmic Equality Property** to "take the log (base 4) of both sides".

Method 3: Use the **Logarithmic Equality Property** to "take the natural log of both sides".

Which method do you like best?

expl 2: Solve.

If you know your powers of 3, it might get you going here.

$3^{4x} = 81$

expl 3: Solve. Try the different methods below.

$1000e^{.09t} = 5000$

Can you isolate the exponential factor on the left side?

Method 1: First divide both sides by 1000 to isolate the exponential factor on the left. Then use the equivalency of $x = a^y$ and $y = \log_a x$ to rewrite the equation in log form.

Method 2: First divide both sides by 1000 to isolate the exponential factor on the left. Then use the **Logarithmic Equality Property** to “take the natural log of both sides”.

Solving Logarithmic Equations: Some equations will need to be simplified using our newly learned log rules in addition to using the exponential and logarithmic properties of this section.

expl 4: Solve. Try the different methods below.

$$\log_3(4w) = 4$$

Method 1: Use the equivalency of $x = a^y$ and $y = \log_a x$ to rewrite the equation in exponential form.

Always check your answers.

Method 2: Use the **Base-Exponent Property** to rewrite this as an exponential equation. Then use the log rules to simplify as you solve for w.

expl 5: Solve.

What is $\log_2 4$?

Work to isolate $\log_2(x-1)$ first.

$$3\log_2(x-1)+\log_2 4=5$$

expl 6: Solve.

$$\log_3(x+14)-\log_3(x+6)=\log_3 x$$

Use your log rules to rewrite the left side as one log. Then use the **Logarithmic Equality Property** to get rid of the logs.

Always check your answers.

You should get into the habit of always checking your solutions in the original equation. Do it now for the previous example.

The methods we use sometimes produce extraneous solutions. So you must check your solutions.

Solving Equations Graphically: As we have seen before, solving an equation graphically is simply a matter of graphing "*y* = the left side" and "*y* = the right side" and seeing where they intersect. One advantage of a graphical solution is that you never get extraneous solutions.

expl 7: Solve using a graphing calculator. Copy the graph here. Do not just TRACE. Use the INTERSECT function on the calculator. Round your solutions to three decimal places.

$$\log_2(x-1)-\log_6(x+2)=2$$

You will need the change of base formula.

expl 8: Solve using a graphing calculator. Copy the graph here. Do not just TRACE. Use the INTERSECT function on the calculator. Round your solutions to three decimal places.

$$2^x-5=3x+3$$

Did you find both solutions?

Worksheet: Using log rules to solve equations:
This worksheet guides you through solutions with step-by-step instructions, providing practice solving equations both algebraically and graphically. It gives good advice on how to graph the pieces of these equations.