We will use our equation solving skills to attack real-life problems.

College algebra
Class notes
Exponential and Logarithmic Applications (section 5.6)

We will see various applications in this section. Keep in mind what the variables are said to represent and you'll be fine. We will, of course, need to use our equation solving skills we have developed through the semester.

Application: Population growth and decay: These problems may refer to a population of people or animals but also money or anything else that grows or decays exponentially. **If a population** (be it a population of a city, or number of deer in a certain area, or money in a bank account) **grows at a fixed rate each year** (or day, or second, etc.), **then it is said to be growing exponentially.** Likewise, if a population decreases by a fixed rate each year (or day, or second, etc.), it is said to be decreasing or decaying exponentially.

Formula for Population Growth and Decay:

We have $P(t) = P_0 e^{kt}$ with the following definitions.

P_0 = initial amount or population (at time 0)

$P(t)$ = amount or population after t years, days, etc.

t = time (years, days, etc.)

k = growth / decay constant

k is specific to situation:
if $k < 0$, decay
if $k > 0$, growth

[You may see $P(t) = P_0 e^{-kt}$ given as the formula for decay (where k is considered to be positive). I think it's easier to use one formula, taking k itself to be positive for growth and negative for decay.]

expl 1: A population of rabbits has an exponential growth rate of 10.5 % per day. Consider an initial population of 100 rabbits.

Use $P(t) = P_0 e^{kt}$ with k in decimal form.

a.) Find the exponential growth function.
b.) Graph the function.
c.) What will the population be after 7 days?
d.) What will the population be after 3 weeks?
e.) Find the doubling time.

What do you think "doubling time" means?

expl 2: United States exports increased from $12 billion in 1950 to $1.550 trillion (or $1,550 billion) in 2009. Assuming that the exponential growth model applies,

a.) Find the value of k and write the exponential growth function. Round k to six decimal places.
b.) Estimate the value of exports, in billions of dollars, in 1972. Round to the nearest tenth of a billion.

We will need to solve an exponential equation.

Application: Half-life of radioactive substances: The half-life of a substance is the amount of time it takes for one-half of the substance to decay.

expl: The half-life of radium is 1690 years. Consider the table below.

t (years)	amount of radium (grams)
0	100
1690	50
3380	25
5070	12.5
6760	6.25

At the beginning of time, we have 100 grams.

Every 1690 years, the amount present decreases by half.

expl 3: Iodine 131 is a radioactive material that decays according to the function $P(t) = P_0 e^{-.087t}$ where P_0 is the initial amount present and $P(t)$ is the amount present at time t (in days). (The decay rate of iodine-131 is 8.7 % per day. That information is already installed in the given formula.) Assume a scientist has a sample of 100 grams of iodine 131.

a.) Graph the function.

b.) How much is left after 9 days?

c.) When will there be 70 grams left?

expl 4: Using the information from the previous question about Iodine 131, find the half-life of Iodine 131.

The half-life is the amount of time for a substance to decay to one-half its original amount. If we started with 100 grams, what should we set $P(t)$ equal to?

expl 5: A fossilized bone has lost 30% of its carbon-14. How old is it? (In other words, how long ago did it die?) Assume the amount of carbon-14 present in animal bones after t years is given by $P(t) = P_0 e^{-0.00012t}$.

Let P_0 be 1. If it has **lost 30%** of its carbon-14, then what is $P(t)$?

Optional Worksheet: Logarithmic and exponential applications: Exponential decay and growth:
This worksheet will provide some guided practice problems. Follow the steps given to work through the problems. These guided problems are followed by more practice problems.

Optional Worksheet: Logarithmic / Exponential applications: Compound interest:
The first part of “Logarithmic / Exponential applications: Compound interest” uses specific functions to understand why the continuously compounding formula works for what we use it for. The second part is composed of guided examples and practice problems for compound interest.