We will learn procedures for finding the x values that make these equations true.

College algebra
Class notes
Solving Rational and Radical Equations (section 3.4)

Algebraically Solving Rational Equations:

Main idea: The most efficient method is to multiply all fractions by their LCD. That eliminates the fractions as you will see. Make sure you always multiply **every term** by the LCD.

expl 1: Solve.

$$\frac{x-3}{x+9}=\frac{6}{x+2}$$

What is the LCD here? So multiply both fractions by that.

Your equation will become quadratic. How do you solve those?

Checking your answer:

An important step in solving rational equations is checking your answer. Sometimes a solution turns out to be what is called an "**extraneous solution**". Even though you get it by doing good algebra, it does **not** make the equation true after all. Remember, the whole point is to find the value(s) of the variable that make the equation true.

Cross-multiplying:

Notice the last problem quickly became $(x + 2)(x - 3) = 6(x + 9)$ once we multiplied by the LCD. We were then able to solve it like any quadratic equation. That step can be short-cut by what is commonly called cross-multiplying.

When you have an equation with just one fraction on each side, you can "cross-multiply" to get a simpler equation with no fractions. But beware! It only works if the equation is in the form "one fraction = another fraction".

expl 2: Solve.

$$\frac{x}{x-1}-\frac{1}{x+1}=\frac{2}{x^2-1}$$

Factor all bottoms to find the LCD.

Remember these?

$$a^2-b^2=(a+b)(a-b)$$
$$a^3-b^3=(a-b)(a^2+ab+b^2)$$
$$a^3+b^3=(a+b)(a^2-ab+b^2)$$

Check your solutions!

expl 3: Solve.

$$\frac{1}{x-6}-\frac{1}{x}=\frac{6}{x^2-6x}$$

When good algebra gets you to an equation that is true no matter what x is, what do you conclude about the original equation?

…But why can't we say the solution is "all real numbers"?

Factor all bottoms to find the LCD.

expl 4: Solve.

$$\frac{7}{2x+6}-\frac{3}{x^2-9}=\frac{5}{x-3}$$

Algebraically Solving Radical Equations:

Main idea: Addition undoes subtraction, division undoes multiplication. What undoes square roots? When your variable is buried under a radical sign, what do you do to unbury it?

What undoes square rooting?

Square root 16 and get 4. What do we do to 4 to get back to the 16?

expl 5: Solve.

$$\sqrt{4x-3}-5=0$$

Isolate the radical first!

Check your solutions!

expl 6: Solve.

$$\sqrt{x+16}+4=x$$

Check your answers whenever you square both sides of an equation.

expl 7: Solve.

$$\sqrt{3x+5}=-7$$

How do you know there is no solution just by looking at the equation?

Would that be the case if it were $\sqrt[3]{3x+5}=-7$?

Literal Equations (Formulas): Consider the formula in the next example. If we know A and P and want to find I, the formula is just what we need. However, if we know A and I and want to find P, this formula can be more work than we want. Imagine if we had many such problems. If we solved the formula for P, it would help out a lot.

expl 8: Solve $I=\sqrt{\frac{A}{P}}-1$ for P.

Isolate P.

expl 9: Solve $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$ for R_2.

Solving equations graphically: The simplest way to solve an equation graphically is to graph y = "left side of equation" and y = "right side of equation" and see where they intersect. Let's take another look at some of the equations we just solved. Draw a quick, labeled graph and plot and label the solutions.

expl 10: Graphically solve.

$$\sqrt{x+16}+4=x$$

Optional Worksheet: Roots and Intersections on your Calculator (82, 83, 85, 86): Here we explore finding roots (commonly called zeroes or x-intercepts) of a single function and the intersection of two functions. We use these skills when we solve equations graphically. Instructions for the 83 will work for the 84 too.