How many outfits can I make with 10 shirts, 5 pairs of pants, and the choice of 3 *swingin'* belts?

General Education Mathematics
Class Notes
Counting: The Fundamental Counting Principle (Section 12.2)

Consider this classic problem.

expl 1: Abby, Bob, Cathy, and Doug are lining up outside of a theatre to buy tickets to a show. Write down several possible ways to line up these four people.

Weren't A, B, C, and D watching a movie in the last section?

How many ways can this line happen? The **Fundamental Counting Principle (FCP)** answers this question easily.

It states that if you have a task (like lining people up outside a theatre) that takes many parts (like 4 places in line), and the first part can be done in a ways, the second part can be done in b ways, the third part can be done in c ways, and so on, then the number of ways you can do the whole task is $a \cdot b \cdot c \cdot \ldots$.

Let's line these four people up, one by one, and we will see how the FCP comes into play. I use the spaces below to organize the problem. Our task has four parts (four places in line) and so we start with four "slots" as the book calls them.

________ ________ ________ ________

1st in line   2nd in line   3rd in line   4th in line

Write in dummy results on top of lines.

Write down how many possibilities for each place in line.

Definition: Factorial: Factorials are a quick way to write the product of any non-negative integer and all the positive integers less than it. For instance, $10! = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$. (If it comes up, we define 0! to be 1.)

Notice the answer to the previous example could be thought of as 4! (pronounced "four factorial").

expl 2: Consider now that we have ten people (let's say they are named A, B, C, D, E, F, G, H, I, and J) waiting at the theatre but only the first four in line will get tickets. How many ways can we assign tickets? Fill in the spaces below.

Write in dummy results on top of lines.

________	________	________	________
1st in line	2nd in line	3rd in line	4th in line

Write down how many possibilities for each place in line.

Here, we see that 10! is *not* the answer because there are *not* ten spaces to fill. We could use the Fundamental Counting Principle (FCP) or we could employ a formula for what is called permutations. We will see permutations (and its close friend, combinations) in the next section.

Let's solve a few more problems with the Fundamental Counting Principle.

expl 3: Three coins (a nickel, a dime, and a penny) and a fair, six-sided die are tossed into the air. We will record whether a heads or tails shows on each coin and the number on the die. (For instance, HHT5 could be a result.) How many different results are possible? Use the spaces below and the Fundamental Counting Principle to help you.

________	________	________	________
nickel	dime	penny	die

expl 4: A menu is reproduced below. A special is underway where a customer can order one appetizer, one entree, and one dessert for the low, low price of $12.95.

Appetizers	**Entrees**	**Desserts**
Clam sampler	Fried Calamari	Chocolate cake
Shrimp cocktail	Lemon Zested Salmon	Cherry pie
Mozzarella sticks	Curried Chicken	
	Beef a la Teriyaki	
	Vegetarian Delight	

a.) How many different ways can this be done?

b.) To get an idea of what we are counting, write out *just three* of the *many* possibilities. You may abbreviate.

expl 5: An algebra test has four multiple-choice questions with five answers (A, B, C, D, E) each, followed by three true-false questions.

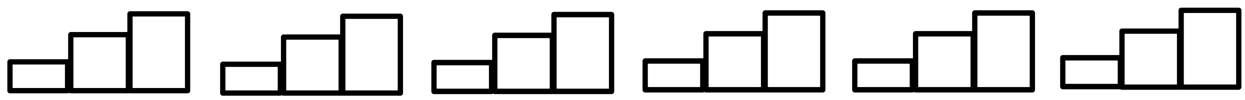
Below are seven blank answer spaces. Fill in a possible answer key for the test described above. Use it to explain your answer on the right.

1. ____
2. ____
3. ____
4. ____
5. ____
6. ____
7. ____

How many answer keys are possible? (A teacher devises one possible answer key when she writes a test. I want to know how many possibilities she has.)

expl 6a: Ten people are competing in a dance competition. How many ways could the top three positions be chosen?

expl 6b. Let's say these ten people are abbreviated as contestants A, B, C, D, E, F, G, H, I, and J. Label the winner's podiums below with six different ways that three of these people can stand there.



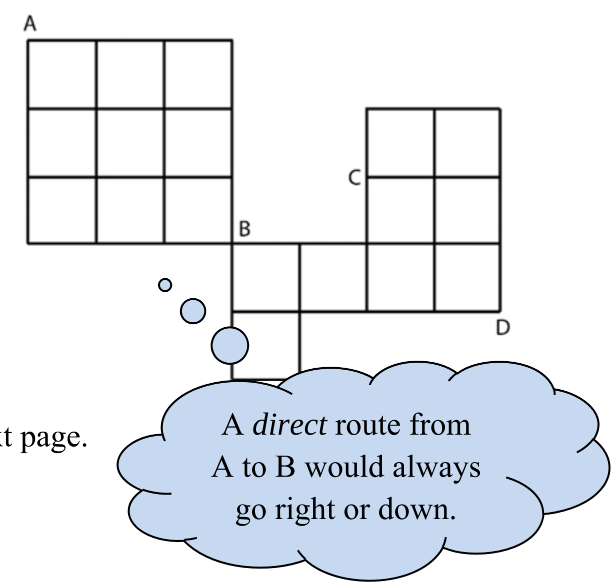
Map and Spatial Problems:

We can use the Fundamental Counting Principle (FCP) in surprising ways. These map problems also have another interesting pattern in store.

expl 7: Let's suppose our city is shown in the map below.

A bicycle delivery-person must ride from Amy's Pumpkin Emporium (A) to Bob's Barbershop (B) to Cecile's Calendar Boutique (C) and then to Derek's Dirt Bikes (D). We are interested in how many *direct* routes are possible. We will work this step-by-step.

We will break this up into three parts, as the FCP says.
1. How many direct routes from A to B?
2. How many direct routes from B to C?
3. How many direct routes from C to D?

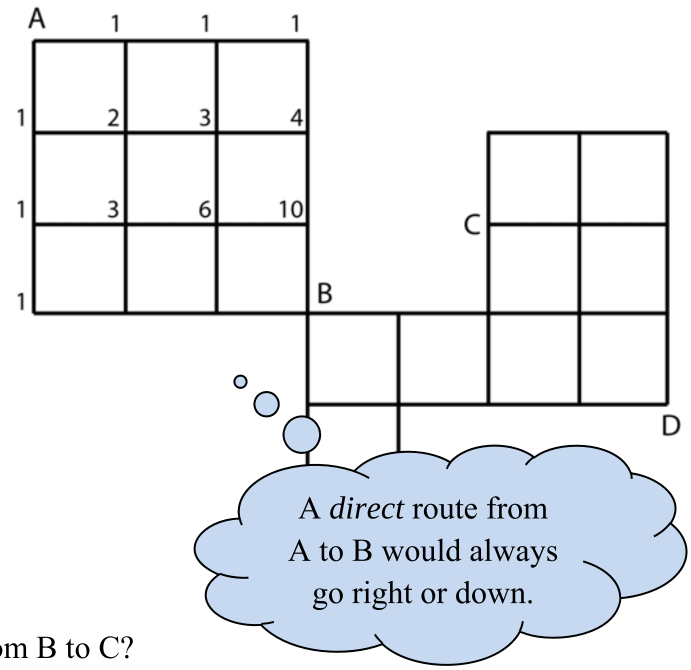


Think on this a bit. We will continue together on the next page.

expl 7 continued: Here, we see a version of the map with numbers on several corners.

For each intersection, the number indicates the number of possible routes from corner A to that intersection. For instance, find the intersection labeled 4 and mark off all direct routes from A to this intersection to verify there are 4 such routes.

a.) Do you see a pattern in these numbers? Can you use it to fill in the bottom row of numbers? So, how many direct routes are possible from A to B?



b.) How many direct routes are possible from B to C?

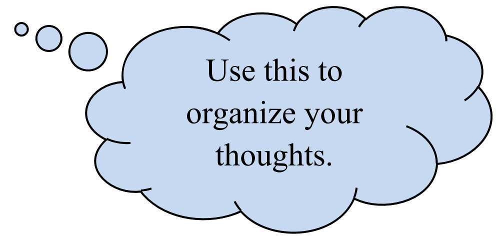
c.) Fill in numbers on the map, similar to those shown for the AB route, to figure the number of routes from C to D. How many direct routes are possible from C to D?

d.) The FCP tells us that the number of ways to do this three-part task is found by multiplying the ways each part can be done. So, how many ways can our intrepid bicycle delivery-person get from A to D?

Problems with Special Conditions:

expl 8: Eight customers in a DMV (Department of Motor Vehicles) are listed as A, B, C, D, E, X, Y, and Z. Customers A, B, C, D, and E are there for license plates renewal. Customers X, Y, and Z are there for their driver's test. How many ways can these people sit in the eight seats available if the following restrictions are put in place?

seat 1   seat 2   seat 3   seat 4   seat 5   seat 6   seat 7   seat 8



a.) *No* restrictions are put in place.

b.) All driver's test applicants must sit in seats 1-3. Can we still use the FCP? How does it change?

c.) All driver's test applicants must sit together in three adjacent seats. Can we still use the FCP? How does it change? Let's explore this together.

i.) How many ways can we choose the three seats for the driver's test applicants?

ii.) Within these three seats, how many ways can these three people be seated?

iii.) How many ways can the remaining five people be seated?

iv.) Oh, exalted Fundamental Counting Principle, do your magic!

When Can the Fundamental Counting Principle *Not* Be Used?:

expl 9: Consider the following problem.

Savvy has four shirts, five pairs of pants, and three *snazzy* belts. She will *not* wear the blue shirt with the red pants, because, … seriously, *Mother*! She cannot fit one belt through the beltloops of the red pants. How many outfits can she make?

Give some reasons we *cannot* use the FCP for this problem. (A tree diagram would be a better tool. Can you make it?)

With and Without Repetition:

expl 10: You are making a six-digit password for your tablet using the digits 0, 1, 2, … , 9. How many possibilities do you have under these conditions?

a.) There are *no* special conditions.

b.) You *cannot* have any repeated digit.

"Without repetition" means if you use 5, you *cannot* again.

c.) The first and last digits *cannot* be 0.

If you *cannot* use 0, how many digits can you choose from?

d.) The first and last digits *cannot* be 0. You *cannot* have any repeated digit.

Worksheet: Supplemental Counting Problems:
Just a few more problems for practice.

Optional Worksheet: The Fundamental Counting Principle and Systematic Lists:
This will give you a solid understanding of the problem with Abby, Bob, Cathy, and Doug lining up outside a theatre that started these notes. There are more practice problems, some of which are repeated in these notes.