Make a conclusion based on examples or algebraic rules. What type of reasoning are you using?

General Education Mathematics
Class Notes
Inductive and Deductive Reasoning (Section 1.2)

There are two major types of reasoning that we use in math and science. These are **inductive reasoning** and **deductive reasoning**.

Definition: Inductive reasoning looks for patterns or examples that share some common characteristic. From the pattern, we conclude some general statement. This conclusion is called a **hypothesis** or **conjecture.**

For instance, look at the string of numbers 2, 4, 8, 16, ___

What would you guess the next number is?

When you use a set of examples or a pattern to make a conjecture, you are using inductive reasoning. This is often described as "going from the specific to the general".

Definition: Deductive reasoning essentially goes the opposite way. It is often described as "going from the general to the specific".

For instance, we know general truths like the properties of real numbers and you can add a number to both sides of an equation and it remains a true equation. Right? Deductive reasoning allows us to be sure of the conclusion when solving an equation like below.

$$\begin{aligned} 4(x+2) &= 24 \\ 4x+8 &= 24 \\ 4x &= 16 \\ x &= 4 \end{aligned}$$

The distribution property simplifies the left side. Then we subtract 8 from both sides and divide both sides by 4. **All of these steps are true for numbers in general.**

We use deductive reasoning when we want to prove, without a doubt, that our conclusion is true. Inductive reasoning arguments can give us a reasonable conclusion but *cannot* guarantee that the conclusion is, in fact, the truth. For instance, perhaps the string of numbers from above (2, 4, 8, 16, ___) were the ages of the people in my household so that the next number is 45?

It may seem underhanded to say that (up above, I was thinking of the number 32 as the next number, were you?) but it points out that inductive reasoning may get you a correct conclusion but you never really *know* for sure. There is *not* enough evidence to say *absolutely* that the pattern you saw was the one the writer had intended. There may be other valid conclusions as well.

expl 1: Use inductive reasoning to predict the next term in the sequences below. Describe the pattern you are using.

a.) 8, 12, 16, 20, 24, _______

Look for a pattern and generalize it to find the next term.

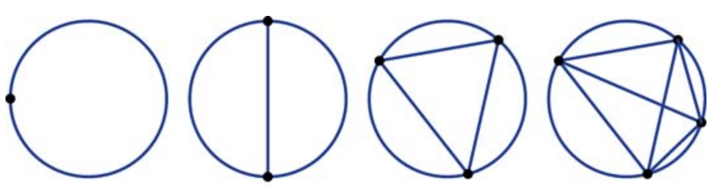
b.) 3, 4, 7, 11, 18, 29, _______

c.) 3, 6, 5, 10, 9, 18, 17, 34, _______

What is this craziness? Hint: Look for a two-step pattern.

Inductive reasoning may seem like a reliable way to reach conclusions, and usually it is. However, there are times when inductive reasoning will lead us astray as mentioned at the bottom of page 1. Let’s look at this next example.

expl 2: We want to divide a circle into regions by selecting points on its circumference and drawing line segments from each point to all the other points. The figure below shows the greatest number of regions that we get if we have one point (no line segment is possible for this case), two, three, and four points.



Number of points	1	2	3	4	5	6
Number of regions	1	2	4	8		

a.) Use inductive reasoning to find the greatest number of regions we would get if we had six points on the edge of the circle. (Look for a pattern in the number of regions in the table.)

expl 2 continued:

b.) Now, draw a circle with six points, connecting them all with one another. Do you get the regions you guessed? (Don't make it too small.)

Inductive reasoning *sometimes* leads to wrong conclusions.

Deductive reasoning, on the other hand, can be used to definitively prove your conclusion. By working from general truths, we can be sure our specific conclusion is correct. Let's work the following puzzle together with deductive reasoning.

expl 3: Four students, Alex, Carmella, Noah, and Winnie, participate in different college activities (debate team, basketball, orchestra, or theater). Use the following clues to determine the activity of each student.

1. Winnie lives in the same apartment complex as the musician and theater participant.
2. The musician and Noah were friends in high school.
3. Carmella has a heavier course load than the basketball player, but fewer credits than the theater participant.
4. Noah, who has the fewest credits, is not on the debate team.

This means Winnie cannot be in orchestra or theater.

Make a table to organize and start eliminating the possibilities.

	debate team	basketball	orchestra	theater
Alex				
Carmella				
Noah				
Winnie				

I usually make an X in a table cell when I know that person could *not* possibly do that activity. Can you eliminate all possibilities except one for each person?

Clue 3 tells us the theater person does *not* have the least number of credits. Combining that with Clue 4 tells us what about Noah?

Worksheet: Magical Math Puzzles:
This worksheet explores some fun little puzzles that use arithmetic to amuse and amaze. You should do questions 1*a* and 2*a* on your own. We will work on the remaining parts of the worksheet together as the process may be new to you.

expl 4: A magic square is a square arrangement of numbers (1 through 16, all used exactly once) such that if you add the numbers in any row, column, or diagonal (from corner to corner), you will always get the same number. Complete the missing numbers in the magic square below.

16	3		13
5			8
	6		12
4		14	

Before we start, this is crucial. The sum of all numbers 1 to 16 is 136. If each row (or column) adds to the same thing, they must all add to 136/4 or 34.

Hint: Start with row 1 or column 4 since they have only one missing number. Use the notion from the thought bubble.

Integers: {…, -3, -2, -1, 0, 1, 2, 3, …}
Even integers greater than 2: {4, 6, 8, 10, 12, …}
First several prime numbers: {2, 3, 5, 7, 11, 13, 17, 19, 23}
Definition of prime number: has no factors other than 1 and itself

Goldbach's Conjecture:
expl 5a: In 1742, Christian Goldbach, a German mathematician who also studied law, conjectured that we could write every even integer greater than 2 as the sum of two prime (*not* necessarily distinct) numbers. Which mathematical statements below *support* his conjecture?

a.) 2 + 12 = 14	b.) 7 + 7 = 14	c.) 3 + 11 = 14
d.) 14 + 12 = 26	e.) 19 + 7 = 26	f.) 2 + 11 + 13 = 26

expl 5b: When you investigate **Goldbach's Conjecture** by looking at examples, are you using inductive or deductive reasoning?

Interestingly, Wikipedia tells me that the conjecture has been shown to hold for all integers less than 4×10^{18}, but remains *unproven* despite considerable effort. Could you find the deductive reasoning argument that would put this 280-year-old conjecture to bed?