All birds lay eggs. A swan is a bird. Therefore, a swan lays eggs.

General Education Mathematics
Class Notes
Logic: Using Euler Diagrams to Verify Syllogisms (Section 3.5)

We were introduced to Euler Diagrams back in the first section of our Logic chapter. Here, we will use them to verify the validity of syllogisms. We did see this term a bit in the last section, but we define it here.

Definition: A **syllogism** consists of a set of statements called **premises** followed by a statement called a **conclusion**. A syllogism is **valid** if whenever its premises are all true, then the conclusion is true. If the conclusion can be false even though all of the premises are true, then the syllogism is **invalid**.

Doesn't that sound just like how we defined an argument? For our purposes, it is. However, if you go looking on the internet, you might find that Aristotle invented the word syllogism to mean a "*valid* argument". That is *not* how we use the word here as we will conclude *some of our syllogisms are invalid*.

The book distinguishes between this section and the last by saying we will see the quantifiers (all, none, some, etc.) in the arguments we study here. Let's jump in!

expl 1: Use an Euler diagram to determine whether this syllogism is valid.

All original parts are under warranty.
This part is an original part.

$\therefore$ This part is under warranty.

Recall that $\therefore$ means therefore.

Draw a large circle and label it "under warranty". How would you draw the circle labeled "original parts"? Does it overlap the first circle? Is it completely within (or outside) the first circle? Place a large dot to represent "This part".

So, can we determine if this syllogism is always valid? Is there any way for the two premises to be true and yet the conclusion be false?

expl 2: Here is another syllogism. Draw the Euler diagram to show it is *invalid*.

All college students are ambitious.
Cameron is ambitious.

∴ Cameron is a college student.

Draw a circle for ambitious and a circle for college students. How are they related? Where do we place Cameron (with a large dot) to show this argument is invalid?

Remember, do *not* let your own opinions factor into it. We are *not* here to argue the merits of the premises. We take those as true. Period. So, you heard it here, all college students are ambitious.

expl 3: For each statement, draw an Euler diagram that shows the stated relationship.

a.) All sharks are ferocious animals.	b.) No sharks are ferocious animals.	c.) Some (but *not* all) sharks are ferocious animals.

How did we differentiate among the statements in our Euler diagrams?

expl 4: There are two possible Euler diagrams for the following set of premises. Can we determine if my spouse is a US president? Can we determine if my spouse is *not* a US president?

Premises: No US presidents have been women.
My spouse is *not* a woman.

We do have to be careful that we are considering *all* possible Euler diagrams. Can you draw them?

expl 5: Determine if this syllogism is valid or invalid.

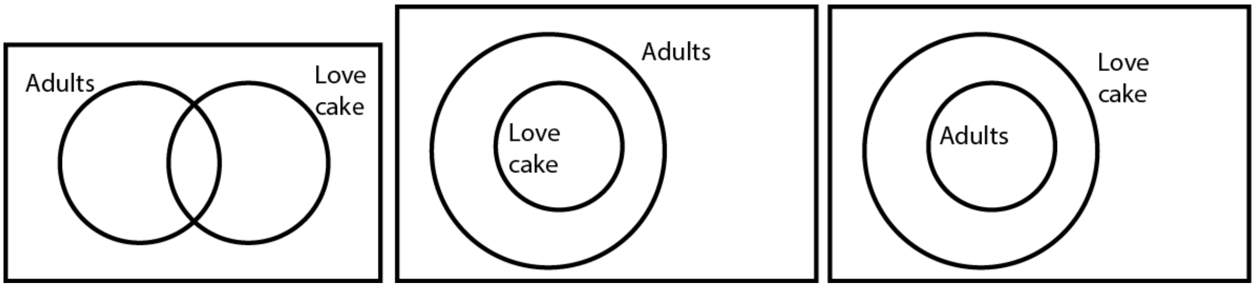
Some adults love cake.
Rachel does *not* love cake.

Therefore, Rachel is *not* an adult.

Try to find an Euler diagram that shows it to be invalid. If we cannot, we will say it is valid.

Euler Diagrams involving the Quantifier "Some":

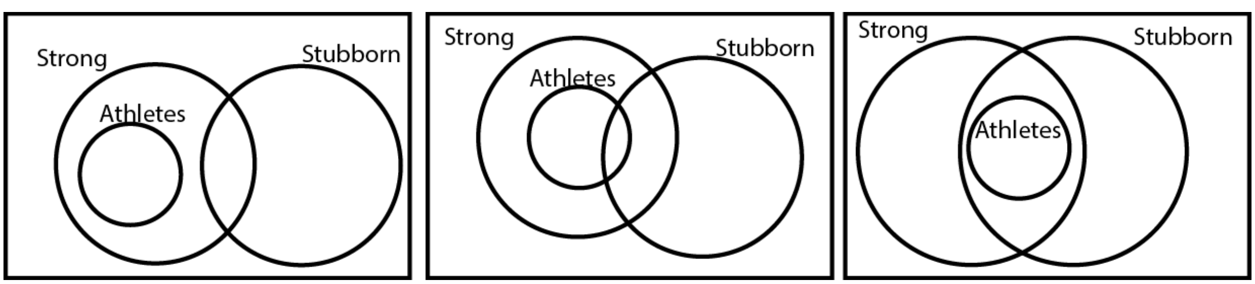
We saw this in the previous example. You probably drew "Some adults love cake" as two overlapping circles. In fact, these three different Euler diagrams could be used to show "Some adults love cake". Shade the area where those adults who love cake are.

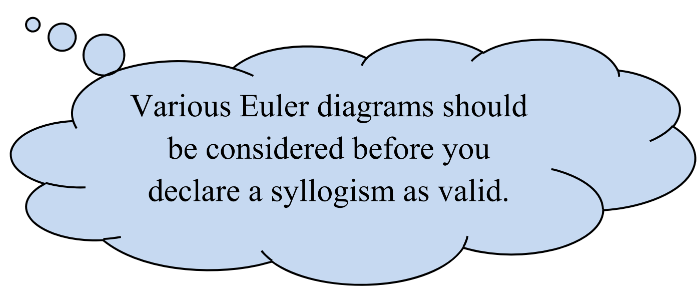


Notice, logically, the statement "Some adults love cake" does *not* rule out the possibility that "All adults love cake". (That is a weird one for me and will take some getting used to.)

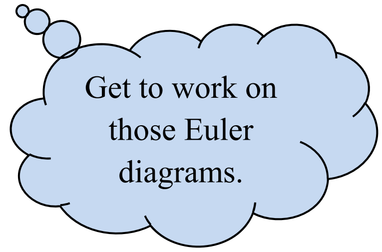
Let's play around with these Euler diagrams a little more.

expl 6: Each Euler diagram below represents the two premises "All athletes are strong. Some strong people are stubborn". (These are *not* the only such Euler diagrams.) For each diagram, state a condition that is shown but has *not* been stated as a premise. There will be more than one correct answer.





expl 7: Complete the syllogism so that it is valid and the conclusion is true. There may be more than one correct answer.



All people who like country music have trucks.
All truck owners have dogs.
Some opera singers like country music.

Therefore, …

expl 8: Choose the valid syllogism for this Euler diagram. Then label the diagram for the syllogism.

a.) All men are monsters.
Sally is a man.

Therefore, Sally is a monster.

b.) Some gnomes are birds.
All birds eat tigers.

Therefore, all gnomes eat tigers.

c.) All crayons are red.
All red people are blue.

Therefore, all blue people are red.

d.) All frogs are telephones.
Murphy is a telephone.

Therefore, Murphy is a frog.