If you pay $1 for a "1 in a 100" chance to win $500, how much, on average, would you expect to win?

General Education Mathematics
Class Notes
Probability: Expected Value (Section 13.4)

Here, we think about games of chance in the long term. If we played a game many, many times, what would we expect to happen? We will see how this can also apply to insurance premiums and business decisions. Let's start with a basic example.

expl 1: Bob has a single (fair, six-sided) die that he will let you roll for $1. If the die comes up as 2 or 4, he will give you $3. Otherwise, you get nothing (except that empty feeling in your pocket where your dollar used to be).

Let's pretend we played this game 100 times and here are our results. What is your average payout? We will fill in this table step by step.

Remember you paid $1 to play.

Die Result	**Number of games (out of 100)**	**Profit or Loss Per Game ($)**	**How much *total* profit or loss for all games?**
2 or 4	30		
1, 3, 5, or 6	70		
Add the profits and losses for all 100 games.			
Divide the sum by 100 to find the average.			

You have 30 wins at $2 each and 70 losses at *negative* $1 each.

So, we found the average "payout" from the game (even when that meant we *lost* $1). Over these 100 plays, we lost more money than we won. So, when we average all 100 payouts, we end up with a *negative* average payout. Bob got the better of us here.

Another way to calculate this is below. Notice the only difference is *when* we divide by this 100.

Expected value = $\$2 \cdot \left({}^{30}\!/_{100} \right) - \$1 \cdot \left({}^{70}\!/_{100} \right) = -\,\$0.10$.

Here, we are multiplying each result by its probability, and then adding those up. This uses a very important formula we will see on the next page.

However, do *not* lose sight of why we started with this example. Expected value simply averages the possible outcomes, taking into account how often each outcome occurs (with its probability). Here, we imagine this game being played 100 times and so used experimental probability. In practice, we will often use theoretical probabilities.

Definition: Expected value: Assume that an experiment has outcomes numbered 1 to n with probabilities $P_1, P_2, P_3, \ldots, P_n$. Assume that each outcome has a numerical value associated with it and these are labeled $V_1, V_2, V_3, \ldots, V_n$. The expected value of the experiment is given by $P_1 \cdot V_1 + P_2 \cdot V_2 + P_3 \cdot V_3 + \ldots + P_n \cdot V_n$.

Take each possible outcome and multiply by its probability. Then add those up.

As example 1 showed us, all we are really doing is averaging the results over time. This is also called a **weighted average**.

You may be given probabilities (as in the next example) or the number of times an outcome has occurred (as in example 1). Note the connection between the number of times an outcome has occurred and its probability. You will need to convert those numbers to probabilities for some problems.

expl 2: The table below shows the breakdown of Netflix customers with respect to how many movies they streamed last month. Find the expected number of movies streamed from Netflix last month.

Netflix Streaming	
Number of movies	**Probability**
0	0.06
1	0.58
2	0.22
3	0.10
4	0.03
5	0.01

So, 58% of the customers streamed 1 movie that month.

The expected value tells us how many movies, on average, we can expect a randomly selected customer to have streamed last month. Be sure to attach the units to your answer above.

Definition: Fair: A **fair game** is one where the expected value is 0. A **fair premium** (paid for insurance) would be one that results in an expected value of 0.

expl 3: Consider the game we saw in example 1. It costs $1 to play and it had an expected value of – $0.10. How much would we have to charge so that this game is **fair**?

We want the average winnings to be $0 instead of – $0.10. Bob could give back 10 cents to everyone each time they play, or charge less for the game in the first place. How much should he charge?

Use the formula to find the expected value with this new fee.

expl 4: An investor has $20,000. She is considering putting her money into a bond with the following known probabilities of return. What would her expected gain or loss be? Fill in the last column of the table to better organize the information and then find the expected value.

Return	**Probability**	**Actual Gain or Loss**
Gain 10%	.40	
No change	.12	
Lose 12%	.48	

What is 10% of $20,000?

Label losses as negative.

Is the expected value negative or positive? Should she invest in this bond?

expl 5: A coffee shop sells pre-packaged bagels but they need help to determine how many they should buy. The manager of the shop has kept track of the number of bagels that were sold on each of the last 30 days, which is shown in the table below.

Demand for Bagels Sold	200	175	150	120
Number of Days with These Sales	5	7	15	3

> For 3 of the 30 days recorded, the shop sold 120 bagels.

The shop buys the bagels for $0.95 and sells them for $1.55. If the manager orders 150 bagels on a certain day, what should she expect to make in profit? Complete the table below to help you organize the facts.

Demand for Bagels Sold	200	175	150	120
Probability the shop could sell this many bagels	5/30	7/30	15/30	3/30
Profit, assuming we have 150 bagels to sell				

> If the demand is more than 150, how many will they sell?

> For each bagel sold, the shop makes $0.60 profit.

> If we sell less than 150 bagels, some must be thrown out. How does that affect profit?

expl 6: Lou's Lightning Insurance offers insurance through a local appliance store. You bought a $2,000 refrigerator and are offered this insurance. If the fridge breaks down, they will replace it. There is a 3% chance that your refrigerator will break down during the insured period. Find a **fair premium** for this insurance, following these steps.

a.) First, consider the case if the insurance costs you nothing. Fill in the table to organize the information and then use the formula for expected value.

Outcome	**Fridge breaks**	**Fridge does *not* break**
Insurance Payout (Net)	$2,000	$0
Probability	.03	

If the probability the fridge breaks down is 3%, then what is the probability it does *not* break down?

Complete: The average insurance policy is worth…

b.) What would Lou charge for the insurance if he wanted the premium to be **fair**? In other words, how much should Lou charge in order to have an expected value of $0? (This would mean the money he makes from all these policies will equal the money he must pay out.)

Insurers do *not* do this. However, they might use this to determine the *lowest* they should charge.

c.) To check, calculate the expected value for this insurance policy using *your* fair premium. Is it really $0?

Outcome	**Fridge breaks**	**Fridge does *not* break**
Insurance Payout (Net)	$\$2{,}000 - x$ =	$-x$ =
Probability	.03	.97

Here, x stands for *your* insurance premium.

expl 7a: Margie has a 52-card poker deck and she will let you pull a card out of the deck for $2. If the card is red or a Queen, then she will pay you $5. If not, she keeps your money. What is the expected value of this card game? Should you play? Fill in the table to organize the information and then use the formula for expected value.

Outcome	**Card is Red or Queen**	**Card is *not* red or Queen**
Payout (Net)	$3	– $2
Probability		

So, what is the probability that the card we choose is red or a Queen? How many successes? How many possibilities?

There are 26 red cards and 2 black Queens. (Two red Queens are part of the 26 red cards.)

expl 7b: Poor Margie! After trying this game on several of her “friends”, she is *not* raking in the money as she thought she would. Why not? If she wants to actually make money on average, what is the minimum she should be charging to play the game?