We start organizing similar things in groups using special notation. To what sets do you belong?

General Education Mathematics
Class Notes
The Language of Sets (Section 2.1)

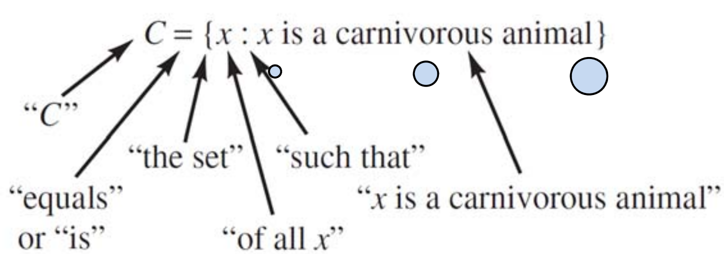
**Definitions:** A **set** will be defined as a collection of objects. The individual objects in a set are called **elements** or **members**.

You might be described as belonging to the set of "humans" or "students". Can you think of other "sets" to which you belong?

We will denote a set by **listing** and **set-builder notations**. The listing method is exactly that, a listing. We will separate elements by commas and put the fancy, curly set brackets around them.

I might list out the set of people in my family as {Stef, Joel, Savvy, Penn, Bat}. I might even call this set F and write F = {Stef, Joel, Savvy, Penn, Bat}.

Set-builder notation gives us a more formal way of denoting sets. It's also good when I cannot list all the elements. Consider the set of all carnivorous animals. Here is the set in set-builder notation. The phrases below show how you say it all; read it from left to right.




Sometimes, you will see a vertical line | instead of the colon shown here.

Notice how it would *not* be easy to write this set in listing notation.

**Definition:** A set is **well-defined** if we are able to tell whether any particular object is an element of that set.

expl 1: Are these sets well-defined?

a.) the set of even numbers

b.) $\{x : x \text{ is tall}\}$

c.) $\{m : m \text{ is an even number and is a letter}\}$

d.) {5, 9, 11, 47, 50}

The set in example 1*c* has *no* elements at all. We will use the concept of a set with no elements quite a lot.

**Definition:** The set that contains no elements is called the **empty set** or **null set.** This set is labeled by the symbol $\varnothing$. Another notation for the empty set is $\{\ \}$.

You should *not* write the empty set as $\{\varnothing\}$. This means the set with that symbol as its only element.

You will see the ellipsis (the … symbol) used in set notation. It is perfectly acceptable to write the set of natural numbers through 100 as {1, 2, 3, …, 100}. However, when you use an ellipsis, make sure the pattern is actually established. I would *not* want to denote the set {5, 9, 11, 47, 50} as {5, 9, 11, …} as no one would see the pattern. (Indeed, there is none since these are the ages of people in my family.)

**Definition:** The **universal set** is the set of all elements under consideration in a given discussion. We often denote the universal set by the capital letter U.

If we were studying the dental habits of teenagers, the universal set might be U = { *x* : *x* is a teenager}. I might then select a few to study and call that set S = {Bobbie, Carol, Tom, Stan, Barbra, Jenny}.

Yeah, I just made those people up.

**Common Sets of Numbers:**

**Natural numbers:** N = {1, 2, 3, …}

**Whole numbers:** W = {0, 1, 2, 3, …}

**Real numbers:** R = {*x* : *x* can be represented on a number line}

Can you draw a number line?

**Integers:** I = {…, -3, -2, -1, 0, 1, 2, 3, …}

**Rational numbers:** Q = {*x* : *x* can be written in the form $a/b$ where *a* and *b* are both integers and *b* is not 0} expl 2: Write each set using listing notation.

a.) The natural numbers that are less than 30 and divisible by 6

b.) The days of the week with eight or more letters

expl 3: Write each set using set-builder notation.

Try to be specific.
Leave no room for
misunderstanding.

a.) {5, 10, 15, 20, …}

b.) The days of the week with eight or more letters

**Elements of a Set Notation:**

We need to be able to talk about the elements in a set. The **notations** $\in$ **and** $\notin$ are used to denote whether an element is in a set or not, respectively. For instance, we will write $6 \in \mathrm{I}$ to mean that "6 is an element of the set of integers".

expl 4: Fill in the missing symbol ($\in$ or $\notin$) to complete the statement.

| | |
|---|---|
| a.) 11 _____ {5, 9, 11, 47, 50} | b.) Mazda ______ {*x* : *x* is a car company} |
| c.) 10 _____ {2, 4, 6, 8} | d.) Apple _______ {*a* : *a* is an actor} |

**Definition: Cardinal number:** The number of elements in set A is called the **cardinal number** or **cardinality** of set A and is denoted $n(A)$. You may see the alternative $|A|$.

A set is finite if its cardinal number is a whole number. An infinite set is one that is *not* finite.

expl 5: Say if each set is finite or infinite. Find the cardinal number of each finite set. Use the $n(A)$ notation, using the appropriate letter for the set.

We will focus on finite sets. However, set theorists do use cardinality to compare the number of elements in infinite sets also.

Does the set of Integers have the same number of elements as the set of Positive Integers?

a.) V = {$x$ : $x$ is a vowel in the English alphabet}

b.) S = {Bobbie, Carol, Tom, Stan, Barbra, Jenny}

c.) T = {1, ½, ¼, 1/8, 1/16, …}

d.) Y = {2, 4, 6}

e.) ∅

What are the elements of this set? How many are there?

f.) P = { {2, 4} , {2, 4, 6}, {2, 4, 6, 8} }

**Worksheet: Sets introduction:**
We will practice the meaning and notation of sets.

expl 6: Find an element of set A that is *not* an element of set B.
A = {$x$ : $x$ is an integer}

B = {$x$ : $x$ is an even number}

Can you find an element of set B that is *not* an element of set A?

**Sets Reference Sheet:**

Complete this list as we proceed through chapter 2. There are extra spots in case you want to add your own items.

| Symbol | Meaning |
|---|---|
| capital letter A | set A |
| $\varnothing$ or { } | empty set |
| U | Universal set |
| $\in$ | is an element of |
| $\notin$ | is *not* an element of |
| $n$(A) | the number of elements in set A |
| A = B | |
| A ≠ B | |
| $A \leftrightarrow B$ | Sets A and B are equivalent sets |
| $A \subseteq B$ | |
| $A \nsubseteq B$ | |
| $A \subset B$ | |
| $A \not\subset B$ | |
| $A \cup B$ | |
| $A \cap B$ | |
| $A'$ or $\overline{A}$ | |
| B – A | |
| $r_1$ | region $r_1$ in a Venn diagram |
| | |
| | |
| | |