> When are two sets the same? How would you define that? How do you know if one set is contained within another?

General Education Mathematics
Class Notes
Comparing Sets (Section 2.2)

Definition: Two sets A and B are **equal** if they have exactly the same members. In this case, we write $A = B$. If A and B are *not* equal, we write $A \neq B$.

expl 1: Tell whether or not the sets E and F are equal. Use the notation $E = F$ or $E \neq F$.

a.) E = {a, b, c, d, g}

F = {b, a, c, d, g}

> Does the order matter?

> Can you think of any elements in one set but *not* the other?

b.) E = ∅

F = {a: a is a number less than 5 and greater than 10}

c.) E = {2, 2, 4, 5, 6}

F = {2, 4, 5, 6}

> Truly, a set should *not* be written with repeated elements.

Definition: Sets A and B are **equivalent**, or in a **one-to-one correspondence**, if $n(A) = n(B)$. Another way of saying this is that two sets are equivalent if they have the same number of elements. We may write $A \leftrightarrow B$.

expl 2: Tell whether or not the sets E and F are equivalent. Use the notation $n(E) = n(F)$ or $n(E) \neq n(F)$.

a.) E = {2, 4, 6, 8, 10}
F = {a, e, i, o, u}

b.) E = {2, 4, 6, 8, 10, 12}
F = {2, 3, 4, 5, …, 12}

Subsets and Proper Subsets:

Definition: The set A is a **subset** of the set B if every element of A is also an element of B. We indicate this relationship by writing $A \subseteq B$. If A is *not* a subset of B, then we write $A \not\subseteq B$.

expl 3: Use the symbols $\subseteq$ and $\not\subseteq$ to fill in the blanks.

a.) {rose, dahlia, daisy} ______ $\{f : f$ is a flower$\}$

b.) {2, 4, 6, 8, 10}______ {2}

c.) I _____ R

d.) {2} _____{2, 4, 6, 8, 10}

e.) 2 _____{2, 4, 6, 8, 10}

Do you remember the sets I and R?

This one is tricky!

By the way, the empty set is considered to be a subset of every set. It is (vacuously) true that every element in the empty set is also an element of any other set.

Definition: The set A is a **proper subset** of set B if $A \subseteq B$ but $A \neq B$. We write that A is a proper subset of B as $A \subset B$. If A is *not* a proper subset of B, then we write $A \not\subset B$.

You can think of the line within the $\subseteq$ (subset) symbol as a (possibly) equal sign. Leaving it off means the two sets could *not* be equal.

expl 4: Tell whether or not these statements are true. Explain your reasoning.

a.) {rose, dahlia, daisy} $\subset$ $\{f : f$ is a flower$\}$

b.) $\{a, e, i, o, u\} \subset \{x : x$ is a vowel of the English alphabet$\}$

Ignore *y*, the wishy-washy bastard!

Venn Diagrams:

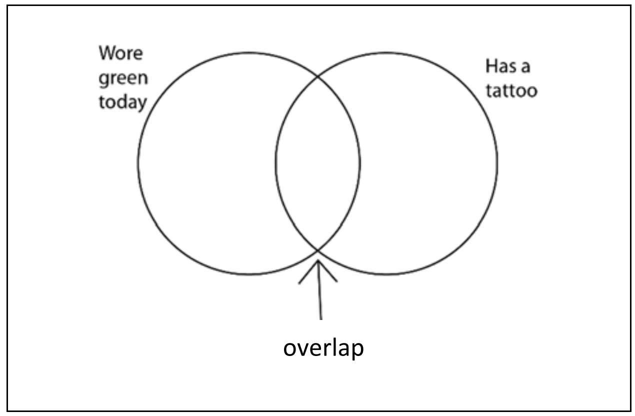
A Venn diagram will visually show the relationships between two or more sets. We will see how we can compare sets and even count the number of elements in each region of a Venn diagram (later).

The Universal set U will be denoted as a rectangle that encompasses all. Each set is represented by a circle. Consider these examples.

Here is a Venn diagram showing two sets of people in my class. Rather than labeling them with letters, I have used phrases.

The left circle represents people who wore green that day. The right circle represents people who have a tattoo.

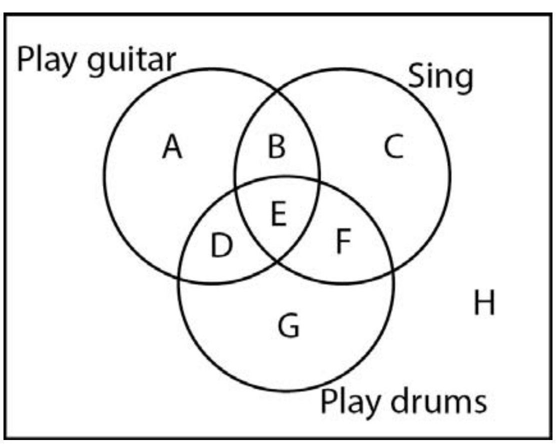
The area where the circles overlap is important. How would you describe that section?



We can do this same thing with three sets. Think about the students at a music school. Perhaps, we are interested in students who play guitar, sing, and/or play the drums.

Can we use Venn diagrams to organize these students? Could we use limited information to figure out how many students play drums but do *not* sing? You betcha!

We will get to these problems later but consider this Venn as an example now.



Just as we saw the overlap above, do you see how these three groups overlap?

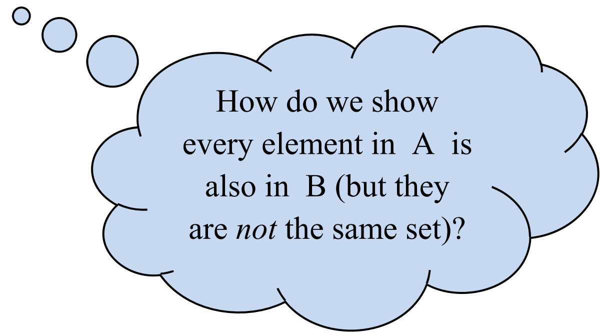
Shade in the region(s) you think represent(s) the students who play drums but do *not* sing.

Where do we find the students who solely play guitar?

What group of students does region E represent?

Usually, a Venn diagram will be drawn so all combinations of overlapping sets are possible. Occasionally, you will see a Venn that aims to show how one set is a proper subset of another set or some other specific relationship.

expl 5: Draw a Venn diagram that would show set A as a proper subset of set B. Include the universal set drawn as a rectangle.



expl 6: Draw a Venn diagram that would show two sets, A and B, that do *not* overlap at all. In other words, there are *no* elements that are in both sets. Include the universal set drawn as a rectangle.

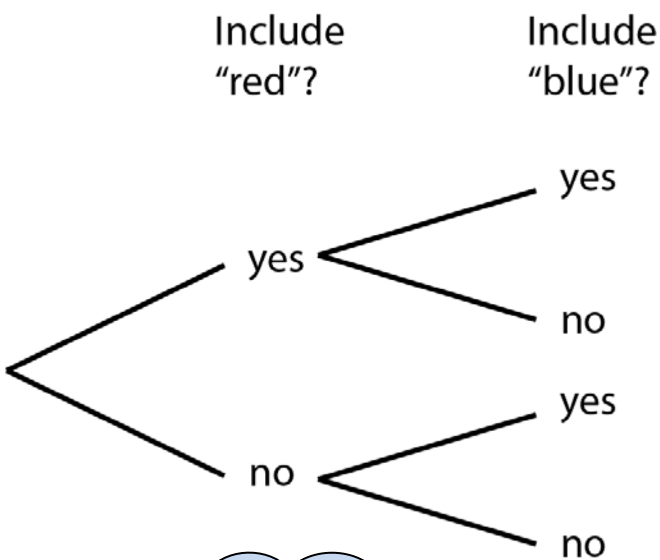
Counting the Number of Possible Subsets of a Given Set (and Recording Them):

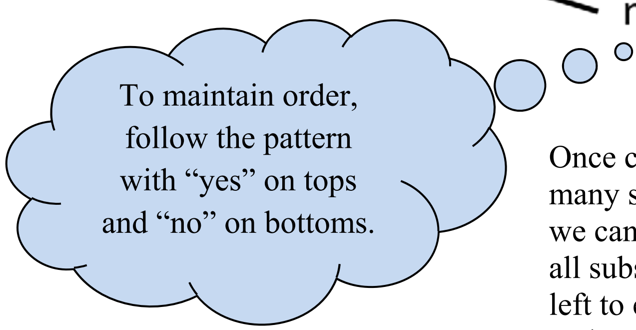
If we want to write out all possible subsets of some larger set, it is nice to know how many there are so we can be confident we have them all. Consider a set with three elements. We'll call this set A = {red, blue, yellow}. We will draw a tree diagram to determine the possible subsets.

The general idea is that we pretend we are forming a subset. We start by asking ourselves if we want to include "red" in the subset. This is a yes or no question, denoted by two branches of our tree on the far left. Above the tree diagram, I have added the labels to help us navigate through.

We continue from each branch end, asking ourselves if we want to include "blue" in the subset. This is a yes or no question, again, denoted by the next branches. Notice, we needed four branches here. That is where my picture ends and your job begins…

Add to the tree diagram, on the right side, more branches that denote us asking ourselves if we want to include "yellow" in the subset. How many branches do you need to draw?





Once complete, the tree diagram not only tells us how many subsets are possible (where is this information?) but we can use the tree diagram to systematically write down all subsets. We do this by following the tree from the far left to each far-right end along each and every possible path.

Do this now and write down all (eight) possible subsets. Notice that the empty set will be included in the list.

Formula for the Number of Possible Subsets:

Of course, we do *not* always want to make a tree diagram. A tree diagram can get cumbersome fast when you have more than three items in a set. So, we do have a formula. But can you guess it? Here are the number of subsets given for a set of various sizes. Can you guess the pattern?

Number of Elements in Set A	**Number of Possible Subsets of A**
1	2
2	4
3	8
4	16
5	32
6	64

Can you describe these 2 subsets? These 4 subsets?

There is a reason the formula is the way it is. It is based on the fact that, as we saw in the tree diagram, every element of the set may or may not be included (two options) in the subset.

Formula for the Number of Possible Subsets:
If a set has k elements, then it has 2^k possible subsets.

Check my table for a set with 6 elements.

expl 7: You are told the following information about the number of students in each major in your class. If we wanted to form subsets, count the numbers of subsets that…

Major	**Number of Students**
Math	7
Nursing	12
Biology	6
Law	5

a.) … only contained math majors?

b.) … contained only math or biology majors?

Give integer answers.