We want to compare two means from dependent samples. Remember we use the *t*-distribution for means.

Statistics
Class Notes
Inference about Two Means: Dependent Samples (Section 11.2)

Treat a group of patients with a new hair loss drug and measure the difference of hair density in each person (before and after). Does the new hair loss drug provide an increase in the *mean* hair density?

A University of Mississippi study tested the reaction times of people, comparing how long it took to press a button upon seeing a red screen versus seeing a blue screen. Is there a difference in *mean* reaction times?

Recall: Determining if Two Samples are Independent:
Definition: A sampling method is **independent** when an individual selected for one sample does *not* dictate which individual is to be in a second sample.

A sampling method is **dependent** when an individual selected to be in one sample is used to determine the individual in the second sample. Dependent samples are often referred to as **matched-pairs** samples. It is possible for an individual to be matched against him or herself.

We will work with the means of matched-pair data.

The procedure we use is the same as we saw when we were analyzing a single mean, except that the **differences** are analyzed.

We will have two (dependent) sets of data. For each matched-pair, we find the difference. The order in which we subtract is important and should *not* be done arbitrarily.

We must verify that the following is true before continuing with hypothesis testing.

- sample data come from simple random sampling or a matched-pairs experiment,
- sample data are dependent (matched pairs),
- sample size is small relative to the population size ($n \le 0.05N$), and
- the differences are normally distributed with no outliers, or the sample size is large ($n \ge 30$).

Small departures from normality will *not* cause trouble. However, outliers are a bigger problem. If outliers exist, do *not* use these procedures.

Summary of the P-value Approach:

Step 1: Determine the null and alternative hypotheses. Again, the hypotheses can be structured in one of three ways:

1. Equal versus *not* equal hypothesis (**two-tailed test**)
   H_0: $\mu_d = 0$
   H_1: $\mu_d \neq 0$

2. Equal versus less than (**left-tailed test**)
   H_0: $\mu_d = 0$
   H_1: $\mu_d < 0$

3. Equal versus greater than (**right-tailed test**)
   H_0: $\mu_d = 0$
   H_1: $\mu_d > 0$

We use μ_d to denote the assumed population mean of the differences.

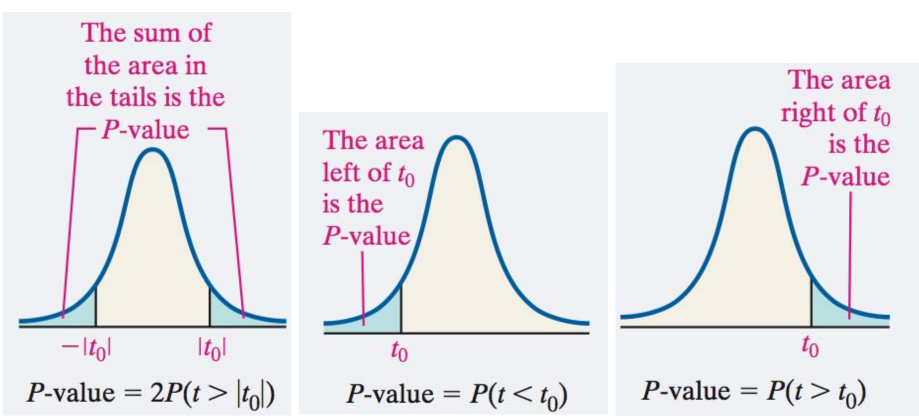
We use $\bar{d}$ and s_d for the mean and standard deviation of the differenced data.

Step 2: Select a level of significance, α, depending on the seriousness of making a Type I error.

Step 3: We *could* compute the test statistic $t_0 = \dfrac{\bar{d} - 0}{\frac{s_d}{\sqrt{n}}} = \dfrac{\bar{d}}{\frac{s_d}{\sqrt{n}}}$ (using $n - 1$ degrees of freedom)

and use Table VII to approximate the P-value. However, we will often use the calculators or StatCrunch to perform the hypothesis testing where this calculation will be done for us.

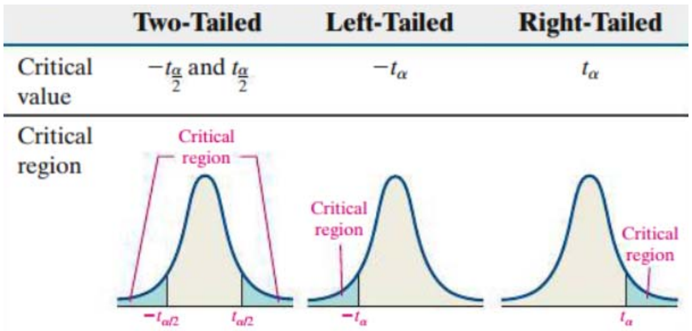
Step 4: If the P-value $< \alpha$, reject the null hypothesis. For an understanding of the P-values, we will look quickly at these pictures.



Step 5: State the conclusion.

Alternatively, Steps 3 and 4 Using Classical Approach:

Step 3: We compute the test statistic $t_0 = \dfrac{\bar{d} - 0}{\frac{s_d}{\sqrt{n}}} = \dfrac{\bar{d}}{\frac{s_d}{\sqrt{n}}}$ which follows Student's t-distribution with $n - 1$ degrees of freedom. Use Table VII to determine the critical value according to the following pictures.



Some homework problems will require the P-value approach while others will ask for critical values (classical approach).

Step 4: Compare the critical value to the test statistic. If the test statistic is in the shaded region shown above for the appropriate test, we reject the null hypothesis.

Recall the instructions to perform a T-Test for the calculator and StatCrunch. We will modify them slightly to accommodate the differenced data.

Instructions for TI Calculators (differs from the book):

1. If needed, enter the differences in **L1**. Just enter the differences (with minus signs) and the calculator will do the math as you go.

2. Press **STAT**. Arrow over the **TESTS**. Select **2: T-Test**.

3. Select **Data** at the top. Press **ENTER**. You'll enter 0 for μ_0, **L1** for **List** and 1 for **Freq**. The final line will give you a spot to tell it you want a two-tailed, left-tailed, or right-tailed test.

Alternatively, before step 2 but after entering the differences in **L1**, run **STAT > CALC > 1: 1-Var Stats** to find the mean and standard deviation of the differences. Then, select **STAT** > **TESTS > 2: T-Test** but this time, select **Stats** at the top. It should fill in the $\bar{x}$ and Sx lines for you. You'll possibly need to enter 0 for μ_0 and the sample size as well as select the correct test (two-tailed, left-tailed, or right-tailed).

4. Finally, select **Calculate** and press **ENTER**. The calculator will output a t-value (the test statistic in step 3 on the previous page), the P-value we need (shown as p), the sample mean and standard deviation, as well as the sample size, presumably to check.

Instructions for StatCrunch:

1. If you have raw data, enter it in the spreadsheet. Of course, coming from MSL homework, click on the overlapping rectangles next to the data, select Open in StatCrunch, and poof!

2. Select **Stat** > **T Stats** > **Paired**.

3. Tell it which columns contain the two samples' data. They will be called **Sample 1** and **Sample 2**. Be aware that $\mu_D = \mu_1 - \mu_2$ and the order is important. Choose the hypothesis test radio button. Enter the value of the mean stated in the null hypothesis (which is 0 in this section) and choose the direction of the alternative hypothesis from the pull-down menu. You can also tell it you want a **QQPlot** (normal probability plot) and a **boxplot**. Click **Compute!**

4. StatCrunch will output the test statistic t_0 (T-Stat) and the P-value. Check the QQPlot to see if it looks linear and the boxplot to see if there are no outliers.

expl 1: A University of Mississippi study tested the mean reaction times of people, comparing how long it took to press a button upon seeing a red screen versus seeing a blue screen. The reaction times were recorded in seconds. Here is the data for a sample of 6 people.

Participant	**1**	**2**	**3**	**4**	**5**	**6**
Blue	0.582	0.481	0.841	0.267	0.685	0.450
Red	0.408	0.407	0.542	0.402	0.456	0.533

a.) Why are these matched-pair data?

b.) The study randomly chose which color a person would be given first. Why should we do that?

expl 1 (continued):
c.) A normal probability plot (QQPlot in StatCrunch) and boxplot of the data indicate the differences are approximately normal with no outliers. Is there a difference in reaction times between blue and red screens? Test the hypothesis at the $\alpha = 0.01$ significance level.

We will use the calculator.

Fill in the various information.

The null hypothesis is _____________ with an alternative hypothesis of _______________.

The test statistic t_0 is ______________ with _____________ degrees of freedom.

The critical value(s) is/are ____.

The P-value is ________________. (Using Table VII, we only get a range of values for the P-value. Technology will outright give us its value.)

We reject / do not reject (select one) the null hypothesis. There is / is not (select one) sufficient evidence at the 0.01 significance level to conclude that there is a difference in the reaction times when blue versus red screens are shown.

Confidence Intervals:
A $(1-\alpha)\cdot 100\%$ confidence interval for μ_d is given by

$$\textbf{Lower bound: } \bar{d} - t_{\alpha/2} \cdot \frac{s_d}{\sqrt{n}} \text{ and } \textbf{Upper bound: } \bar{d} + t_{\alpha/2} \cdot \frac{s_d}{\sqrt{n}}$$

where $t_{\alpha/2}$ is the critical value with $n-1$ degrees of freedom.

expl 2: Construct a 99% confidence interval for the differenced data used in example 1. Interpret the answer. Does it match with the result from the hypothesis test?

Instructions for Calculator:
Do the same as for hypothesis tests, except select STAT > Tests > 8:TInterval. Tell it the confidence level.

Instructions for StatCrunch:
It is the same as before. Enter data and select **Stat** > **T Stats** > **Paired**.

Again, the setup is the same but we choose the Confidence interval radio button. Enter the level of confidence. Click **Compute!**

If you like to do stuff by hand, Table VII is provided here.

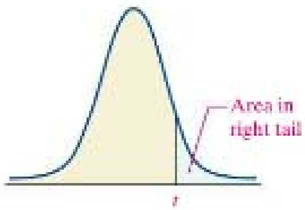


Table VII

Degrees of Freedom	0.25	0.20	0.15	0.10	0.05	0.025	0.02	0.01	0.005	0.0025	0.001	0.0005
	t-Distribution Area in Right Tail											
1	1.000	1.376	1.963	3.078	6.314	12.706	15.894	31.821	63.657	127.321	318.309	636.619
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.089	22.327	31.599
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.215	12.924
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.610	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
31	0.682	0.853	1.054	1.309	1.696	2.040	2.144	2.453	2.744	3.022	3.375	3.633
32	0.682	0.853	1.054	1.309	1.694	2.037	2.141	2.449	2.738	3.015	3.365	3.622
33	0.682	0.853	1.053	1.308	1.692	2.035	2.138	2.445	2.733	3.008	3.356	3.611
34	0.682	0.852	1.052	1.307	1.691	2.032	2.136	2.441	2.728	3.002	3.348	3.601
35	0.682	0.852	1.052	1.306	1.690	2.030	2.133	2.438	2.724	2.996	3.340	3.591
36	0.681	0.852	1.052	1.306	1.688	2.028	2.131	2.434	2.719	2.990	3.333	3.582
37	0.681	0.851	1.051	1.305	1.687	2.026	2.129	2.431	2.715	2.985	3.326	3.574
38	0.681	0.851	1.051	1.304	1.686	2.024	2.127	2.429	2.712	2.980	3.319	3.566
39	0.681	0.851	1.050	1.304	1.685	2.023	2.125	2.426	2.708	2.976	3.313	3.558
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
70	0.678	0.847	1.044	1.294	1.667	1.994	2.093	2.381	2.648	2.899	3.211	3.435
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
90	0.677	0.846	1.042	1.291	1.662	1.987	2.084	2.368	2.632	2.878	3.183	3.402
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z	0.674	0.842	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.090	3.291