Picture ten people. How many ways can we hand out a prize to seven of them?

General Education Statistics
Class Notes
Counting Techniques: Combinations and Permutations (Section 5.5)

The book covers more than we will here. We will only cover the counting techniques that we will be using in future chapters.

We are interested in counting things like how many five-card poker hands are possible from a deck of 52 cards. Perhaps we have three pople lined up and want to know how many ways two of them could have O-negative blood.

Consider this classic problem.

expl 1: Abby, Bob, Cathy, and Doug are lining up outside of a theatre to buy tickets to a show. Write down several possible ways to line up these four people.

How many ways can this line happen? The **Fundamental Counting Principle** (aka **Multiplication Rule of Counting**) answers this question easily.

It states that if you have a task (like lining people up outside a theatre) that takes k parts (like 4 places in line), and the first part can be done in p ways, the second part can be done in q ways, … and continuing to the end, the k^{th} part can be done in v ways, then the number of ways you can do the whole task is $p*q*.....*v$.

So, our task has four parts (four places in line). I use the spaces below to organize the problem.

________	________	________	________
1st in line	2nd in line	3rd in line	4th in line

Write in dummy results on top of lines.

Write down how many possibilities for each place in line.

Definition: Factorial: Factorials are a quick way to write the product of any non-negative integer and all the positive integers less than it. For instance, $10! = 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1$. (If it comes up, we define 0! to be 1.)

Notice the answer to the previous example could be thought of as 4! ("four factorial").

expl 2: Consider now that we have ten people (let's say they are named A, B, C, D, E, F, G, H, I, and J) waiting at the theatre but only the first four in line will get tickets. How many ways can we assign tickets? Fill in the spaces below.

Write in dummy results on top of lines.

________	________	________	________
1st in line	2nd in line	3rd in line	4th in line

Write down how many possibilities for each place in line.

Here, we see that $10!$ is *not* the answer because there are *not* ten spaces to fill. We could use the Fundamental Counting Principle or we could employ a formula for what is called permutations.

Permutations of n things taken r at a time:
The number of ways we can arrange r items chosen from n total, distinct items is given by $_{n}P_{r} = \dfrac{n!}{(n-r)!}$. Here, we say that the order matters. For instance, the line-up GBCE is different than the line-up BCEG. Also, as in the theatre examples, repetition of items is *not* allowed.

A person cannot occupy two spaces in line.

Try out this formula to verify you get the same as you got in the previous example.

Calculator: MATH > PRB

Let's change the scenario up a bit.

expl 3: Let's say we have those ten people (A, B, C, D, E, F, G, H, I, and J) in a room. We want to select four of them to receive a prize. It does *not* matter what order the people are chosen; all four will get the same prize. Write down several possible groups of four people.

Would CEGA be different from GAEC?

How many ways can we do this? Permutations will *not* help us here as the order of the four people does *not* matter. Instead, we use combinations.

Combinations of n things taken r at a time:
The number of ways we can group r items chosen from n total, distinct items is given by $_nC_r = \frac{_nP_r}{r!} = \frac{n!}{r!(n-r)!}$. Here, we say that the order does *not* matter. For instance, the line-up GBCE is *not* different than the line-up BCEG. Also, as in the theatre examples, repetition of items is *not* allowed.

A person cannot occupy two spaces in a group.

Optional worksheet: Permutations and Combinations:
This worksheet works on these two similar but different concepts and how they are related. That discussion is followed by some practice problems.

expl 4: A football kicker will make ten attempts at the goal (through the uprights). How many ways can she make seven of those goals?

Choose seven of the ten attempts to be “made”.

She could make the first seven kicks, missing the rest. She could make, make, make, miss, miss, miss, make, make, make, make. …

expl 5: A health clinic has three patients in the waiting room. How many ways can exactly two of them have O-negative blood? List out the possibilities with the people being denoted “O-neg” or “not”.

We will choose 2 of the 3 patients to be O-negative.

expl 6: A health clinic has twenty patients in the waiting room.

a.) How many ways can exactly seven of them have O-negative blood?

Why do we use combinations?

b.) How many ways can seven or eight of them have O-negative blood?

c.) How many ways can exactly twenty of them have O-negative blood?