How do we know that our random variable is normally distributed?

General Education Statistics Class Notes

Assessing Normality (Section 7.3)

We use these great statistics to answer all sorts of questions about a random variable. But how can we be sure that we are really dealing with a *normal* distribution. That assumption underlies all we have done recently. We will investigate normal probability plots.

**Definition:** A **normal probability plot** plots observed data versus **normal scores**.

A **normal score** is the expected *z*-score of the data value if the distribution of the random variable is normal. The expected *z*-score of an observed value will depend upon the number of observations in the data set.

Here are the instructions for making this plot. However, we will *not* be doing this by hand. Rather, we will interpret the output given by the calculator or StatCrunch.

## **Drawing a Normal Probability Plot:**

**Step 1** Arrange the data in ascending order.

**Step 2** Compute  $f_i = \frac{i - 0.375}{n + 0.25}$  where *i* is the index of each data value (its position in the list) and *n* is the number of observations. This  $f_i$  is the expected proportion of observations less than or equal to the  $i^{th}$  data value.

**Step 3** Find the *z*-score corresponding to  $f_i$  from Table V.

**Step 4** Plot the observed values on the horizontal axis and the corresponding expected *z*-scores on the vertical axis.

Once you plot those points, what do you do?

## **Assessing a Normal Probability Plot:**

If sample data is taken from a population that is normally distributed, a normal probability plot of the actual values versus the expected *z*-scores will be approximately linear.

We'll perform linear regression on this plot. If the linear correlation coefficient is greater than the critical value found in Table VI in the book, then we can conclude that the data could come from a population that is normally distributed.

Which is great since now we are justified in doing the stats we have been doing.

expl 1: A random sample of weekly work logs at an automobile repair station was obtained, and the average number of customers per day was recorded. Draw a normal probability plot and

determine if	the population	is normally	distributed.
			_

Averag	ge Numb	er of Cus	tomers i	n a Day		Let's use
26	24	22	25	23	<u> </u>	StatCrunch
24	25	23	25	22		here.

## **Instructions for Normal Probability Plots on StatCrunch:**

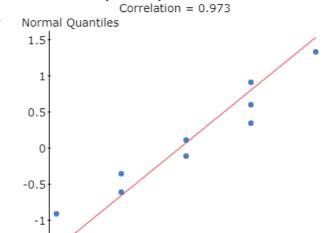
Enter the raw data in column var1.

Select **Graph** > **QQ Plot**. Use **var1** as the Selected Column. Check the boxes to add the *Correlation statistic* and *Normal quantiles on y-axis*. You can further add labels for the *x*- and *y*-axes and title your graph. Click **Compute!** when you are ready.

Now that we have this plot and the correlation coefficient, let's do some work ourselves.

Look up the critical value in Table VI for the sample size. This table is reproduced here in part.

Table VI for Assessing Normality				
Sample Critical size, n Value				
5 5	0.880			
10	0.918			
15	0.939			
20	0.951			
25	0.959			
30	0.960			



24

Average Number of Customers

26

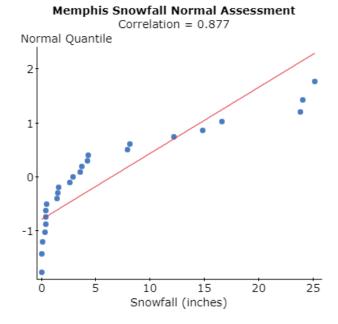
Auto Repair Shop Normal Assessment

As the sample size gets larger, the plot needs to be more linear for the population to be considered normal.

Compare the correlation coefficient to the critical value. If the correlation coefficient is greater than the critical value, then we say the population is normal. What is your conclusion?

expl 2: A random sample of 25 years between 1890 and 2011 recorded the amount of snowfall, in inches, for Memphis, TN. Below is the data. Look at the normal probability plot provided. What is your conclusion about the normality of this population? Explain.

Inches of Snowfall in a Year					
24.0	7.9	1.5	0.0	0.3	
0.4	8.1	4.3	0.0	0.5	
3.6	2.9	0.4	2.6	0.1	
16.6	1.4	23.8	25.1	1.6	
12.2	14.8	0.4	3.7	4.2	



In fact, let's look at a histogram (from StatCrunch) of the data. Does the data look normal?

