Functions: Composition and operations Solutions NAME:

This worksheet will provide practice for adding, subtracting, dividing, and composing two functions. We will also investigate domains.

1. Let $f(x)=\sqrt{5x+7}-4x$ and $g(x)=5x^2+7x-5$. Perform the desired operations. Simplify your answers.

a.) $f(x)+g(x)$

$$=\sqrt{5x+7}-4x+5x^2+7x-5$$
$$=\sqrt{5x+7}+5x^2+3x-5$$

b.) $f(x)-g(x)$

$$=\sqrt{5x+7}-4x-\left(5x^2+7x-5\right)$$
$$=\sqrt{5x+7}-4x-5x^2-7x+5$$
$$=\sqrt{5x+7}-5x^2-11x+5$$

c.) $\dfrac{f(x)}{g(x)}$

$$=\frac{\sqrt{5x+7}-4x}{5x^2+7x-5}$$

2. Let's think about the domain of the three functions of question 1.

a.) What is the domain of $f(x)=\sqrt{5x+7}-4x$? (Remember, we would try to find x values that would make us square root negative numbers or divide by zero. There is no division here, so only worry about the square root part. We'll exclude such x values and what's left over is the domain.)

> *We cannot take the square root of a negative number so solve* $5x+7\geq 0$ *to find the x-values that will work in the function. The term "4x" plays no part since it would not cause division by zero or square roots of negative numbers. The domain is* $x\geq -7/5$ *or* $[-1.4, \infty)$.

b.) What is the domain of $g(x) = 5x^2 + 7x - 5$? (Remember, we would try to find x values that would make us square root negative numbers or divide by zero. There are no divisions or square roots here. So what's the domain?)

all real numbers

c.) The domain of both $f(x) + g(x)$ and $f(x) - g(x)$ will be the numbers that are in **both** the domains of *f* and *g*. What is the domain of $f(x) + g(x)$ and $f(x) - g(x)$?

$x \geq -7/5$ *or* [-1.4, ∞)

d.) The domain of $\dfrac{f(x)}{g(x)}$ are the numbers that are in both the domains of *f* and *g* and also, do not make *g* zero. If that happens, then we'd be dividing by zero and we can't do that. What's the domain of $\dfrac{f(x)}{g(x)}$? (Solve the equation $0 = 5x^2 + 7x - 5$ to find the values that make *g* zero.)

We start with $x \geq -7/5$ *or [-1.4,* ∞*) but we need to also exclude the values that make the bottom zero. The values that make* $0 = 5x^2 + 7x - 5$ *are x = -1.92 and .52. So the domain is [-1.4, .52) and (.52,* ∞*). Another way to say this "all real numbers greater than or equal to -1.4, excluding .52". (Notice the -1.92 did not play a role since it was already excluded from [-1.4,* ∞*).)*

3. Let $f(x) = -3x + 7$ and $g(x) = 2x^2 - 8$.

a.) Find $f(g(x))$. Simplify.

$$f(g(x)) = f(2x^2 - 8) = -3(2x^2 - 8) + 7 = -6x^2 + 24 + 7 = -6x^2 + 31$$

b.) Find $g(f(x))$. Simplify.

$$\begin{aligned} g(f(x)) = g(-3x+7) = 2(-3x+7)^2 - 8 &= 2(9x^2 - 42x + 49) - 8 \\ &= 18x^2 - 84x + 98 - 8 \\ &= 18x^2 - 84x + 90 \end{aligned}$$

4. The number of cars N (per day) produced at a factory after t hours of operation is given by $N(t) = 100t - 5t^2$ where t varies from 0 to 10. If the cost C (in dollars) of producing N cars is $C(N) = 15000 + 8000N$, find the cost as a function of time. In other words, find the relationship between t hours of operation and the cost. Simplify your answer.

$$C(N(t)) = C(100t - 5t^2) = 15{,}000 + 8{,}000(100t - 5t^2)$$
$$= 15{,}000 + 800{,}000t - 40{,}000t^2$$
$$C_2(t) = 15{,}000 + 800{,}000t - 40{,}000t^2$$

This formula gives us the cost if we input the number of hours of operation. I called it $C_2(t)$ to denote that it's different than the original cost formula.

Let's check the answer from above. We'll find the cost of running the factory 9 hours per day two different ways.

a.) Use your formula from above to find the cost of running the factory 9 hours per day.

$$C_2(t) = 15{,}000 + 800{,}000t - 40{,}000t^2$$
$$C_2(9) = 15{,}000 + 800{,}000 * (9) - 40{,}000 * (9)^2$$
$$= 15{,}000 + 7{,}200{,}000 - 3{,}240{,}000$$
$$= 3{,}975{,}000$$

If the factory operates for 9 hours, it will cost the factory $3,975,000.

b.) Now, use the given $N(t)$ formula to find the number of cars they would produce in 9 hours.

$$N(t) = 100t - 5t^2$$
$$N(9) = 100 * (9) - 5 * (9)^2$$
$$= 495$$

If the factory operates for 9 hours, they will produce 495 cars.

c.) Put your answer to part b into the given $C(N)$ formula to find the cost of producing that many cars. Does this match your answer to part a?

$$C(N) = 15000 + 8000N$$
$$C(495) = 15{,}000 + 8{,}000(495)$$
$$= 3{,}975{,}000$$

So if they operate for 9 hours, they will make 495 cars. If they make 495 cars, it will cost them $3,975,000.

One cool thing about composition is that it eliminates the middle step. You go straight from hours of operation to cost, without figuring the number of cars in the middle. Part a above combines the two parts b and c into one step.

5. Let $f(x) = 2x - 5$ and $g(x) = .5(x+5)$. Show that $f(g(x)) = x$ and $g(f(x)) = x$.

$$\begin{aligned} f(g(x)) &= f(.5(x+5)) \\ &= 2(.5(x+5)) - 5 \\ &= x + 5 - 5 \\ &= x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(2x-5) \\ &= .5(2x - 5 + 5) \\ &= .5(2x) \\ &= x \end{aligned}$$

Follow the order of operations to make sense of it. You also need to be good on functional notation. On the left, we've shown that f undoes g. On the right, we've shown that g undoes f.

6. Let $f(x) = \dfrac{4x^2+5}{3}$ and $g(x) = -3x + 7$. Find the following.

a.) $f(g(-3))$

$$g(-3) = -3(-3) + 7 = 16$$

$$f(g(-3)) = f(16) = \frac{4(16)^2+5}{3} = \frac{1029}{3} = 343$$

b.) $g(4) + f(4)$

$$g(4) = -3(4) + 7 = -5$$

$$f(4) = \frac{4(4)^2+5}{3} = 23$$

$$g(4) + f(4) = -5 + 23 = 18$$

c.) $f(g(x))$

$$f(g(x)) = f(-3x+7) = \frac{4(-3x+7)^2+5}{3} = \frac{4(9x^2-42x+49)+5}{3}$$

$$= \frac{36x^2 - 168x + 196 + 5}{3} = \frac{36x^2 - 168x + 201}{3}$$

d.) the domain of $f(g(x))$ [HINT: It's probably easiest to simply look at the formula for $f(g(x))$ and ask yourself "Are there any real numbers that make me divide by zero or square root negative numbers?" Exclude those from the domain.]

There are no x values that would make us divide by zero or square root negative numbers. So the domain *is all real numbers.*