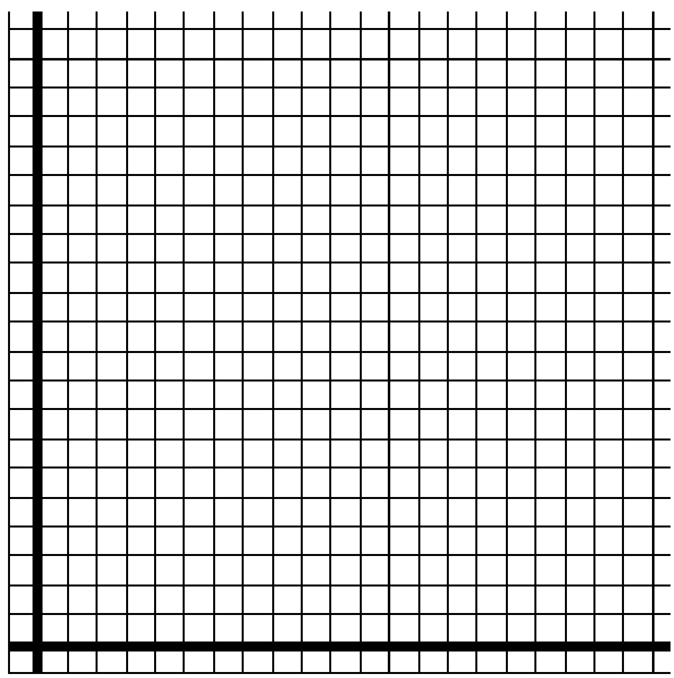
Math 127 Precalculus NAME:
Sequences and Series

Part One:
Go to www.math.ilstu.edu/saolear.
Click on the Problems button on the left.
Click on Limits and the Bouncing Ball. Read the screen and consider this problem.

1. Create a table for the number of bounces and the height of the ball. Use at least six points.

2. Graph these as points (x,y) where the number of the bounce is x and the height of the ball is y. Label the scale of your graph. (Technically, since we are only using the integer values for x, we do not connect this collection of points.)



3. Look at your table of values. What pattern do you notice? Can you write an equation that relates x and y?

4. If the initial height were 100 or 1,000,000 feet, how would the equation change?

Part Two:
For the remainder of the ditto, use an initial height of 20 feet.

The relationship between the bounce number and the height of the ball is a **sequence**. All this really means is that the domain values (the x's) are all positive integers, so we can talk of the 1^{st}, 2^{nd}, 3^{rd}, and so forth terms of our sequence. We might write the sequence as $\{a_n\} = \{ 20(^5/_6)^{n-1} \}$. This means that the n^{th} term is $20(^5/_6)^{n-1}$. (This assumes you used (0, 20) as your first point and we call it a_1.) We then can write $a_1 = 20$, $a_2 = 16.67$, $a_3 = 13.89$ and so forth.

In fact this sequence is a **geometric sequence**. This is because each term is determined by multiplying the previous term by some constant, called the **common ratio**. What is the common ratio in this sequence?

Part Three:
If we wanted to add the total distance the ball fell (not including the distance it takes to bounce back up), what would the sum look like? Write it once with the actual numbers and then write each term as $20(^5/_6)^{n-1}$ substituting the proper number for n.

How do you know where to stop? Theoretically, this goes on forever. Hence this is an **infinite geometric series**. So you should have written a few terms down to establish a pattern and then put "…." after it.

Instead of writing it as $20(^5/_6)^0 + 20(^5/_6)^1 + 20(^5/_6)^2 + 20(^5/_6)^3 + 20(^5/_6)^4 + \ldots$, we could use **shorthand summation (or sigma) notation** and write $\sum_{k=1}^{\infty} 20\left(5/6\right)^{k-1}$. We read this as "the summation of $20(^5/_6)^{k-1}$ from k equals 1 to k equals ∞." This means, for the first term we substitute 1 for k getting $20(^5/_6)^{1-1}$ or $20(^5/_6)^0$. For the second term we substitute 2 for k getting $20(^5/_6)^1$. For the third term we substitute 3 for k getting $20(^5/_6)^2$. The sigma in front tells us to add these terms. Verify for yourself this is the same as the longhand sum.

Since the common ratio of our sequence has an absolute value less than 1, we can use the formula on page 877 to find the **sum of this infinite geometric series**. What are *r* and *a* in our sequence? Use the formula to find the sum.

Now what if we wanted only to sum the distances that were shown on the movie. Recall the last height recorded was 1.87 feet. We shall use this for a place to stop. So now we want $\sum_{k=1}^{14} 20\left(5/6\right)^{k-1}$. Notice the top number is now 14, not infinity. This means the last term will be when k = 14.

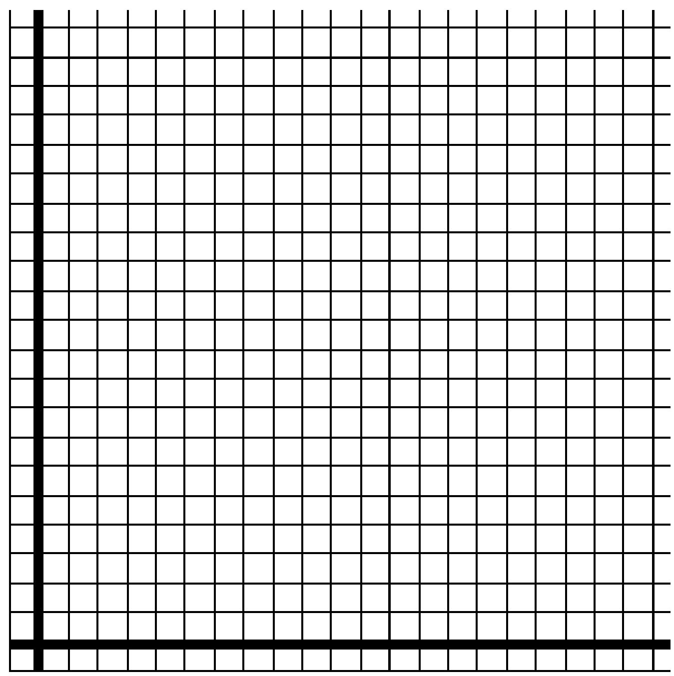
Locate the formula for the **sum of the first n terms of a geometric sequence** on page 874. Use the formula to find the sum of the first fourteen terms of our sequence.

Part Four:
To be taken home and thought about. Bring back on Wednesday.

Now let's ponder the question of why the formula for the sum of an infinite series (pg 877) would not work if the absolute value of r was greater than or equal to one.

If $|r| = 1$, we also have $r = 1$. Notice this makes the denominator zero and so cannot be done. But why does the formula not work if r is 2 or 5?

Try it out. Make up a geometric series where the common ratio is greater than one. Write it down using notation similar to $\{a_n\} = \{\ 20(^5/_6)^{n-1}\ \}$. Graph the points (n, a_n) where n is the number of the term and a_n is the term itself. Label the scale of your graph.



Now why do you think we cannot find the sum of this infinite series?