Simple and compound interest NAME:

These terms are the same whether you are the borrower or the lender, but I describe the words by thinking about borrowing the money.

**Principal:** initial amount you borrow; sometimes called **present value**
**Interest rate:** amount you are charged for the right to borrow the money; usually given as a percentage of the amount borrowed, like 8%
**Future value:** the amount you owe (including the principal and the interest) in the end; sometimes called **maturity value**

**Simple interest:**
Let's say we borrow $100 at an 8% annual interest rate. That means the interest we are charged will be 8% of the original amount for each year we keep the money. Let's say we pay back the money after 1 year. The interest is found by multiplying $.08 * 100$, (that's 8% of $100). This means the interest for one year is $8. For each year I keep the money, I owe another $8.

Simple interest problems use the equation $I = P * r * t$ where *P* is the principal, *r* is the annual interest rate (in decimal form), *t* is the number of years, and *I* is the total interest that is charged.

1. Complete the table for borrowing $100 at 8% annual interest. The one thing that changes through the table is the amount of time I take to pay it back (*t*).

| **Interest earned for *P* = $100, *r* = .08 (or 8%)** | |
|---|---|
| **Number of years (*t*)** | **Total interest earned ($I = P * r * t$)** |
| 1 | |
| 2 | |
| 5 | |
| 14 | |

Now, let's think about the total amount you owe and will need to be paid back. After 1 year, I need to pay back $108, which is the principal plus the interest. After 14 years, I need to pay back $212. That's $100 that I originally borrowed and $8 for each of the 14 years I kept the money.

The formula we will use to find the total amount owed is $A = P + P * r * t$ . That's the principal plus the interest. This is often written as $A = P(1 + rt)$. (We factored *P* from both terms using the distribution property to get this form.) The variable *A* is called the **future value** because it's how much the $100 is worth (to the lender or bank) in the future, after *t* years.

2. Use the formula $A = P(1 + rt)$ to find the total amount owed for each of the following values of *t*. Again, we will use $100 for *P* and .08 for *r*.

| **Total amount owed for *P* = $100, *r* = .08 (or 8%)** | |
|---|---|
| **Number of years (*t*)** | **Total amount owed ($A = P(1 + rt)$)** |
| 1 | |
| 2 | |
| 5 | |
| 14 | |

Notice these answers are just the principal ($100) plus the interest we found in the table for question 1.

3. Practice the simple interest formula using different values of *P*, *r*, and *t*. Make sure you convert *r* to decimal form.

| **Total amount owed for *P* dollars, *r* interest rate, *t* years for a <u>simple interest</u> loan** | | | |
|---|---|---|---|
| **Principal (*P*)** | **Annual interest rate (*r*)** | **Number of years (*t*)** | **Total amount owed ($A = P(1 + rt)$)** |
| $250 | 7% | 4 | |
| $50 | 6.5% | 10 | |
| $700 | 5% | 3 | |
| $20,000 | 14% | 5 | |

**Compound interest:** Compound interest is more complicated. Suppose we borrow \$100 at 8% interest, compounded yearly. This means, that after the first year, we owe \$108. But if I keep the money for another year, the interest for the second year is found by taking 8% of the \$108, not the original amount I borrowed. The interest is compounded, meaning you are charged for interest not only on the original \$100, but also on the previous interest.

4. Let's say we deposit \$100 in a bank account which pays 8% interest, compounded yearly. Complete the table to find the amount at the beginning of each year and the amount at the end of each year. (The amount at the beginning of the year will be the amount at the end of the previous year. Just carry that number down to the next row.) Round your answers to two decimal places.

| **Amount in account after *t* years (original investment = \$100, *r* = .08)** | | |
|---|---|---|
| ***t* (years)** | **Amount at beginning of year (\$)** | **Amount at end of year (Multiply amount at beginning of year by 1.08)** |
| 1 | 100 | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |

Notice the amount after 5 years is different than what it was when we were using simple interest. Look at the table in question 2 to see what the \$100 was worth after 5 years using simple interest.

This table suggests that the formula $A = P(1+r)^t$ would tell us the future value (*A*) of money we invest (*P*) at the annual interest rate (*r*), compounded yearly for *t* years.

Unfortunately, there are more details. Usually, the interest is not compounded every year, but maybe every three months, or every month, or every day. That complicates the formula we need a bit. We will use the formula $A = P\left(1+\frac{r}{m}\right)^n$ where *A*, *P*, and *r* are what we're used to, *m* is the number of times it compounds in a year, and *n* is the total number of times it compounds over the life of the loan. So *n* could be thought of as the number of times it compounds per year times the number of years, *mt*.

If you solve this equation for *P* by dividing both sides by the parentheses stuff, you get $P = \dfrac{A}{\left(1+\frac{r}{m}\right)^n}$. You might use it if you are asked for *P* instead of *A*. I do not do this; I always use the first formula and solve appropriately if I am asked to find *P*.

Let's do a few problems to practice this formula.

5. I loan my brother $500, at an annual interest rate of 4%, compounded monthly, for 5 years. How much will he have to repay?

> *Since this is compounded monthly, we will use the compound interest formula,* $A = P\left(1+\frac{r}{m}\right)^n$. *I am told the original amount, P, is 500. The annual interest rate, r, is 4% or .04. If it compounds monthly, that's 12 times a year so m is 12. My brother will keep the money for 5 years. If it compounds 12 times during each of the 5 years, it compounds a total of 60 (5 times 12) times. So n is 60. Putting the numbers where they go, we get* $A = 500\left(1+\frac{.04}{12}\right)^{60}$. *Watch the order of operations here. Simplify the stuff inside the parentheses to get* $A = 500(1.0033)^{60}$. *Then evaluate the exponent part to get* $A = 500 * 1.22$. *Lastly, multiply to get the amount my brother owes me after 5 years is $610.50. Round intermediate answers to at least four decimal places to get better accuracy in your final answer.*

6. I borrow $1500 from the bank at a 7.5% annual interest rate, compounded four times a year, for 10 years. How much will I have to repay?

7. I took out a loan that was compounded daily (365 times a year) at 4% interest for 5 years. The total amount I repaid was $1500. How much was loaned to me? (Here, notice we were given $A$ and are looking for $P$. Put the information into the formula and then solve for $P$ by dividing by the parentheses stuff. Alternatively, use the formula that is already solved for $P$.)

8. A CD pays 3% interest, compounded quarterly (four times a year). If I deposit $500 and leave it there for 3 years, how much money will I have at the end of 3 years?

9. You will invest money now in an account that pays 7% compounded monthly so that you’ll have $5,000 in 5 years. How much must you invest now?

**Continuously compounding interest:**
Instead of the interest compounding every year, or every month, think of the interest compounding every second, or even quicker than that. Immediately after the interest is added in, it's compounded again and again and again. This is the idea of continuously compounding interest. The formula we will use for this situation is $A = Pe^{rt}$ where $P$ is the principal, $r$ is the annual interest rate (in decimal form), and $A$ is the amount after $t$ years. The letter $e$ is the irrational number approximately equal to 2.72.

Let's practice the formula.

10. I invest $500 in an account that pays 6% interest, compounded continuously. How much will there be in the account after 10 years?

> *Since it's compounded continuously, we use the formula* $A = Pe^{rt}$ *. Here,* $P$ *is the amount I invest or $500. The interest rate (in decimal form) is .06. The value of t is given to be 10. Our formula becomes* $A = 500e^{.06*10}$ *. Find the "e^x" button on your calculator or use 2.72 in place of e. Make sure you multiply the exponent out before you do any other operation. This gives us* $A = 500e^{.6}$ *or $911.40. (This calculation used 2.72 as a rounded value for e.)*

11. I invest $1200 in an account that pays 3% interest, compounded continuously. How much will there be in the account after 4 years?

12. Suppose my savings account pays 5% interest, compounded continuously, and I save my money in the account for 6 years. How much would I have to deposit in order to have $1000 at the end of the 6 years?