Properties of real numbers NAME:

This worksheet will try to make the properties of real numbers more meaningful and memorable. We will use them a lot during the semester. Having them firmly in your head will make algebra easier.

Definition of a real number
Our class only deals with real numbers. The real numbers are essentially every number you've seen so far in life except the imaginary (or complex numbers) such as $7 + 3i$ or $\sqrt{-5}$.

Real numbers include fractions (or rational numbers), zero, negatives, and even irrational numbers like $\sqrt{2}$ or π.

Definition of an integer
An integer is a number in the set {…,-3, -2, -1, 0, 1, 2, 3,…}. We will refer to integers many times during the semester. We will also talk about **non-negative integers**; they are composed of the positive integers and zero.

Closure of real numbers over multiplication and addition
This property makes algebra work. It says if I take two real numbers and multiply, add, subtract, or divide them, I'll still have a real number when I'm through. This makes it possible to say that if x is a real number, then $\frac{4x^2+5x}{2x-3}$ is also a real number. The real numbers are said to be closed under addition, multiplication, and subtraction. (The real numbers are actually not closed under division. There is one real number that when we divide by it, you do **not** end up with another real number. Do you know which it is?) This is important because as we deal with expressions like $\frac{4x^2+5x}{2x-3}$, we have to remember that all it is, is a real number.

We know a lot about real numbers and how they behave. To understand algebra, we have to somehow transfer that knowledge to algebraic expressions that represent real numbers.

This worksheet will help us investigate many properties of real numbers. We will explore a property using actual numbers, and then we look at how it is used with variables.

Factoring

Any real number can be written as a product of its factors. For instance, $45 = 5 * 9$. This allows us to reduce fractions such as $\frac{45}{10}$. We factor the top and bottom of the fraction, and cancel common factors: $\frac{45}{10} = \frac{5*9}{5*2} = \frac{5}{5} * \frac{9}{2} = \frac{9}{2}$. This allows us to mean exactly $\frac{45}{10}$, but write it more simply as $\frac{9}{2}$. Let's practice a couple before we move to algebra.

Simplify the fractions by factoring the top and bottom completely and canceling common factors like in the example above. Write it out explicitly like the above example so you internalize what is happening.

a.) $\frac{28}{48}$

b.) $\frac{60}{75}$

c.) Because expressions such as $4x^2y$ are real numbers, they are also factorable. What are the four factors of $4x^2y$? List them with commas.

d.) Simplify the following algebraic expression. Notice the common factor of $4xy$ on top and bottom; factor both top and bottom and cancel the common factor. Write it out explicitly so you internalize what is happening.

$$\frac{4x^2y}{8xy^3}$$

e.) Simplify the following algebraic expression. Notice the common factor of $7ab^2$ on top and bottom; factor both top and bottom and cancel the common factor. Write it out explicitly so you internalize what is happening.

$$\frac{35a^5b^2}{14ab^4}$$

Distribution property $a*(b+c)=a*b+a*c$

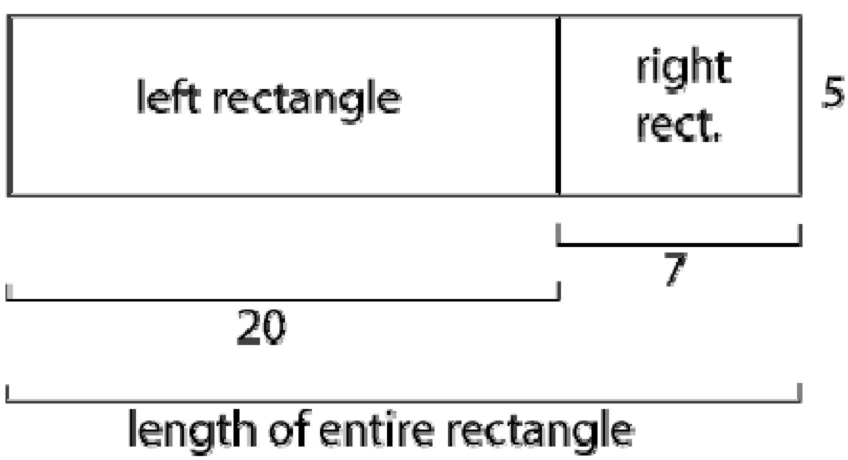
To help understand how the distribution formula works, let's play with the rectangles below.

The area of a rectangle is its length times its width. For the three rectangles below, find

1.) the area of the left rectangle = _______ x _______ = _______,

2.) the area of the right rectangle = _______ x _______ = _______, and

3.) the area of the entire rectangle = _______ x _______ = _______.



What do you notice about the answers from above? How can you justify what happens with the distribution property? Use the distribution property to write an equation concerning the areas found above.

Let's work through some examples of the distribution property using actual numbers. For each side of the equation, work it out using the order of operations to simplify it. Notice this shows the equation is true. I'll work the first for you to show you what to do.

a.) $4*(3+6)=4*3+4*6$

Left: Do parentheses first.

left side: $4*(3+6)=4(9)=36$

right side: $4*3+4*6=12+24=36$

Right: Do multiplications first.

b.) $5*(10+3)=5*10+5*3$

left side:

right side:

c.) $2*(13-7)=2*13-2*7$

left side:

right side:

Notice the distribution property lets us write an expression two different ways:

1. as the sum of two terms with a common factor (right side above), or
2. as the product of two factors (left side above).

For the following sums, find the (largest) common factor between the two terms and use the distribution property to rewrite it as two factors. The first one is done for you. Notice the sum and the product you'll end up with are equivalent.

a.) 10 + 15

$= 5 * 2 + 5 * 3$

$= 5 * (2 + 3)$

$= 5 * 5$

Notice the two terms, 10 and 15, have a common factor of 5.

b.) 33 + 21

c.) 36 – 54

Let's try it with a few variables. Remember, the variables represent real numbers, so by closure these expressions are just real numbers. We can factor and simplify algebraic expressions just like we do with real numbers. Notice in each example below, there is a common factor between the two terms. Factor it out with the distribution property.

a.) $3x^2 + 6y$

b.) $4ab^2 + 2a$

c.) $5xy^2 - 20xyz$

Use what you have learned to simplify the following. Use the distribution property to factor the top, and then cancel common factors from top and bottom.

a.) $\dfrac{3x^2 + 6y}{3x}$

b.) $\dfrac{4ab^2 + 2a}{6a^2}$

c.) $\dfrac{5xy^2 - 20xyz}{y - 4z}$

Commutative property of real numbers over multiplication and addition

$a * b = b * a$

$a + b = b + a$

$4 * 5 = 5 * 4$
$4 + 5 = 5 + 4$

These properties tell us that the order does not matter when we multiply or add two numbers. We use this quite a lot, although often we do not specifically denote it.

What operations are not commutative? Think of two numbers. Subtract them in both directions. Do you get the same result? Do the same for division. Would you say subtraction and division are commutative? Write down evidence of your experimentation.

Associative property of real numbers over multiplication and addition

$a + (b + c) = (a + b) + c$

$a * (b * c) = (a * b) * c$

Let’s verify these rules with actual numbers. For each side of the equation, work it out using the order of operations to simplify it. Notice this shows the equation is true. I’ll work the first for you to show you what to do.

a.) $4 + (5 + 7) = (4 + 5) + 7$

Do parentheses first.

left side: $4 + (5 + 7) = 4 + 12 = 16$

right side: $(4 + 5) + 7 = 9 + 7 = 16$

b.) 12 + (3 + 6) = (12 + 3) + 6

left side:

right side:

c.) $3 * (6 * 2) = (3 * 6) * 2$

left side:

right side:

d.) $-2 * (7 * 10) = (-2 * 7) * 10$

left side:

right side:

Combining like terms and the distribution property

Notice the terms of $3x^2 + 4x^2$ have a common factor of x^2. If we factor that out, we get $3x^2 + 4x^2 = (3+4)x^2 = 7x^2$. This is what we know as combining like terms but notice it is just the distribution property. Rewrite and simplify the following expressions. The first one is done for you.

a.) $6x + 4x$

$= (6+4)x$ Factor out the *x*.

$= 10x$

b.) $5a^2 + 7a^2$

c.) $10y^2 - 5y^2$

d.) $3x^2 + 8x^2 + 4y + 6y$

e.) $12xy + 4xy - 3x^2 + 7x^2$

Practice
Simplify the following expressions. State the property or properties that contribute to each step. The first one is done for you.

a.) $\frac{x^2}{y} * \left(\frac{10+5x+7x+6}{4xy}\right)$

$$\frac{x^2}{y} * \left(\frac{10+5x+7x+6}{4xy}\right)$$

Commutativity of addition, combining like terms

$$= \frac{x^2}{y} * \left(\frac{16+12x}{4xy}\right)$$

Distribution property

$$= \frac{x^2}{y} * \left(\frac{4(4+3x)}{4xy}\right)$$

Canceling common factors

$$= \frac{x^2}{y} * \left(\frac{(4+3x)}{xy}\right)$$

Multiplying fractions

$$= \frac{x^2(4+3x)}{xy^2}$$

Canceling common factors

$$= \frac{x(4+3x)}{y^2}$$

b.) $5ab^3 * \left(\frac{6a^2 - 8a}{10a^2b^3} \right)$

c.) $\frac{2}{yz} * \frac{3xyz^2 - 4xz}{2}$

d.) $(5a + 6a)\frac{12xy^2}{3x}$

e.) $\left(\dfrac{4xy^2 + 2x}{6y^2 + 3}\right)^2$

f.) $4(3x^2 + x - 5) + x^2 + 2x$

g.) $2(3xy + 4x^3 + 7xy) - 9xy - 2x^3$