Exploration of ASS (or SSA) congruency

Query: Suppose you are given two side lengths and a non-included angle's measure for a triangle. Does this information determine a unique triangle? If it does, we can conclude that if two separate triangles have two pairs of congruent corresponding sides and a non-included congruent angle, then the triangles can be called congruent.
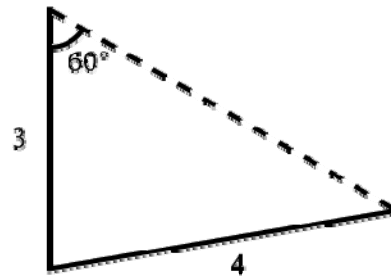
1. In class, we said it did not. There are nuances involved here that were not discussed in class. Sometimes the criteria **does** determine a unique triangle. The first paragraph below shows an example of that.

Sometimes the criteria **does not** determine a unique triangle. The second and third paragraphs below show an example of how that might occur. The fourth paragraph explains how I came up with my example.
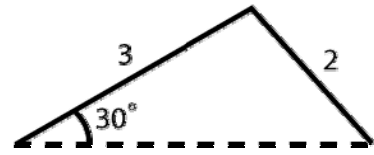
2. Consider the criteria that the two sides are 3 and 4 units long and the non-included angle is 60 degrees. These criteria actually do determine a unique triangle. I drew it to the right.

In class, we said that another triangle could be formed if we rotated the side of length 4. However, it would have to become longer or shorter to meet up with the third side of the triangle. This would create another triangle but the side lengths would no longer be 3 and 4. Any other manipulation would also change the criteria with which we started.
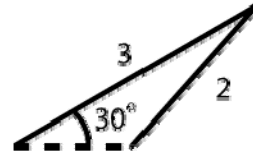
This is why when I tried to rotate the side of length 4 on the board, the picture just looked wrong. I assumed this was due to my poor drawing but that was not the case.

3. Consider the criteria that the two sides are 3 and 2 units long and the non-included angle is 30 degrees. Here is one possibility.
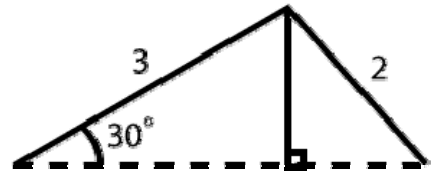
Here is another possibility.

4. I drew this second possibility by drawing the altitude of my first triangle and then reflecting the side of length 2 over this line.

To the right, I have drawn the first triangle with its altitude. Now imagine reflecting the side of length 2 over this line to form another possibility for this side's position. That creates the second possibility for this triangle. Why do you think that works?

5. The selection of the side lengths and angle measure play a huge role in determining if the triangle is unique or not. I had to use the Law of Sines to delve deeper into this quandary. Very cool!