Permutations and Combinations NAME:

1.) There are five people, Abby, Bob, Cathy, Doug, and Edgar, in a room. How many ways can we line up three of them to receive 1st, 2nd, and 3rd place prizes? The spaces might help you organize your thoughts.

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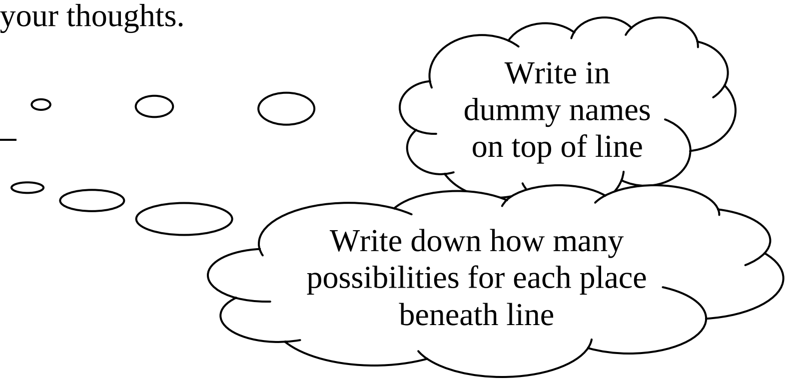

2.) Below are the many different ways (60 total) we could line up 3 of these 5 people to receive the prizes. (They are in order of 1st, 2nd, and 3rd places.)

ABC	ABD	ABE	ACB	ACD	ACE
ADB	ADC	ADE	AEB	AEC	AED
BAC	BAD	BAE	BCA	BCD	BCE
BDA	BDC	BDE	BEA	BEC	BED
CAB	CAD	CAE	CBA	CBD	CBE
CDA	CDB	CDE	CEA	CEB	CED
DAB	DAC	DAE	DBA	DBC	DBE
DCA	DCB	DCE	DEA	DEB	DEC
EAB	EAC	EAD	EBA	EBC	EBD
ECA	ECB	ECD	EDA	EDB	EDC

These 60 possibilities are what we call **permutations**. In fact, these are the "permutations of 5 things, taken 3 at a time". This means you have a total of 5 things (people) to choose from and you are selecting 3 of them to line up in a row.

The formula for the number of "permutations of *n* objects taken *r* at a time" is $\frac{n!}{(n-r)!}$.

This is denoted by $P(n, r)$ or $P\binom{n}{r}$ or ${}_nP_r$. Using our value of *n* and *r*, show the formula gets us the same answer as we got in question 1.

Let's move on to the idea of **combinations** and take the place orders out of the situation. We'll count how many different groups of three people there are, not how many different ordered lists (1^{st}, 2^{nd}, and 3^{rd}) there are. Follow the reasoning below.

3.) How many of these 60 permutations have the same three people in them? For example, how many groups of the 60 possibilities have Abby, Bob, and Cathy and no one else? Circle these possibilities in the listing above. Considering only this group of 3 people, how many ways can they be arranged in 1^{st}, 2^{nd}, and 3^{rd} places? (This should match the number of possibilities you circled.) How might you justify this number with the Fundamental Counting Principle? Write the number of groups that have Abby, Bob, and Cathy and no one else in factorial form.

4.) Each set of 3 people can be arranged 3! or 6 ways. So if we want to count only the number of groups of 3 we can make out of these 5 people (and not think of this as an ordered list with 1^{st}, 2^{nd}, and 3^{rd} places but rather as just a set of 3 people), we need only count each group of 3 people once. Instead of writing ABC, ACB, BAC, BCA, CAB, and CBA, we would only count one of these groups, say ABC.

This is the idea of **combinations**. Instead of counting the number of ways that we could line 3 people up in order (1^{st}, 2^{nd}, and 3^{rd} places), we want to count the number of different groups of 3 people.

Since each group of 3 people appears 3! or 6 times in the total list of $\frac{5!}{(5-3)!} = 60$ possibilities, we should divide 60 by 6 to get the number of distinct groups (combinations). This is essentially what the formula for combinations does. Below are the ten different combinations. Notice no two groups share the same three people.

ABC	ABD	ABE	ACD	ACE
ADE	BCD	BCE	BDE	CDE

The formula for the number of "combinations of *n* things, taken *r* at a time" is $\frac{n!}{r!(n-r)!}$.

This is denoted by $C(n, r)$ or $C\binom{n}{r}$ or ${}_nC_r$. Use this formula to verify the number of combinations of our 5 people, taken 3 at a time. Again, notice this tells us the number of different groups of 3 people.

The main difference between permutations and combinations is that permutations take order into account. Combinations do not. We saw that if we want to count the number of permutations, we would count groups such as ABC, ACB, BAC, BCA, CAB, and CBA as six different possibilities. On the other hand, when we want to count combinations, all six of those groups are considered to be the same, and we would write down only one, say ABC. **When order matters, we use permutations. When order does not matter, we use combinations.**

Let's practice determining which is needed and how to use the formulas. Determine if the problem calls for permutations or combinations and then use the appropriate formula to find the answers. Parts *a* and *b* of each question should point out the difference between combinations and permutations.

5a.) There are 10 people in a contest. If prizes are awarded for 1st, 2nd, and 3rd places, how many different ways can this done?

5b.) There are 10 people in a knitting group. If 3 are to be chosen to attend a conference, how many ways can this be done?

6a.) There are 52 cards in a poker deck. If a poker hand has 5 cards in it, how many possible poker hands are there in a deck?

6b.) There are 52 cards in a poker deck. I will draw one card at a time until I have 5 cards and record both the card and the order in which it was drawn (1st, 2nd, 3rd, 4th, 5th). How many ways can this be done?

7a.) A rare bird dealer has sixteen distinct birds and four cages. The cages are four different colors, yellow, blue, red, and gold. If she wants to put one bird in each cage, how many ways can she do this? (There will be some birds without cages. That's okay. They will be free as a …well, bird.)

7b.) A rare bird dealer has sixteen distinct birds and a cage that holds four birds. If she wants to put four birds in this cage, how many ways can she do this?

8a.) Twenty horses are in a race. Prizes will be awarded for 1^{st}, 2^{nd}, 3^{rd}, 4^{th}, and 5^{th} places. How many ways can this be done?

8b.) Twenty horses are in a race. A prize of $100 will be awarded to each horse that comes in the top five. How many ways can this be done?

9a.) There are four contestants in a race, Amy, Becky, Chris, and Devon. Find the number of ways that two of them can be selected to win 1st and 2nd places. Use their initials to write out all of these possibilities.

9b.) From the four people listed above, two will be selected to attend a conference on good sportsmanship. Here, it makes no difference if Amy is chosen, and then Becky or if Becky is chosen, and then Amy. Find the number of ways we could choose two people to attend the conference. In your above list, cross out the possibilities that are repeats if we want only the combinations.

10.) Write two problems like what you have seen here, one using permutations and one using combinations. Explain why the first needs permutations and the second needs combinations to solve.