Exploration of Independence concerning Number 10 on Probability Worksheet 3

Number 10 on Probability Worksheet 3 reads

> Consider a deck of poker cards. A poker deck contains four suits: diamonds, hearts, spades, and clubs. The diamonds and hearts are red and the spades and clubs are black. Each suit has thirteen cards: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King. This makes a total of 52 cards. A face card will be defined to be a Jack, Queen, or King. I will select one card from the deck. What is the probability that it is a red face card? (We can find this probability by finding "number of successes divided by number of possibilities" getting $^6\!/_{52}$. However, let's find it by considering the two events "card is red" and "card is face".)
>
> Mutually exclusive? _______
>
> Independent? _______

At first glance, the events "card is red" and "card is face" appear to not be independent since they refer to the same card. In this case, we would find

$$\begin{aligned}
&\text{P(card is red AND card is face)} \\
&= \text{P(card is red)} * \text{P(card is face given card is red)} \\
&= \left(\frac{26}{52}\right)\left(\frac{6}{26}\right) \\
&= \frac{6}{52}
\end{aligned}$$

Notice, this makes sense since there are 6 successes in a deck of 52 possibilities.

However, the two events can be shown to be independent. We see this by showing P(card is face given card is red) is equal to P(card is face). If these two probabilities are equal, then the event "card is red" would not affect the probability of the event "card is face". Therefore, the two events would be independent.

We see that P(card is face given card is red) is $\frac{6}{26}$ because there are 26 possibilities for the card (since it must be a red card) and there are 6 successes (6 of these cards are face cards). Also, P(card is face) = $\frac{12}{52}$ or $\frac{6}{26}$ because there are 52 possibilities and 12 successes (12 face cards in a deck). Therefore, these probabilities are equal and the two events are independent. Whether or not the card is red does not affect the probability that it is also a face card.

So the formula P(card is red) $*$ P(card is face given card is red) is the same as P(card is red) $*$ P(card is face) when the two events are independent. So using the first formula does not affect the answer adversely.

This only works this way because of the symmetry of the deck. Let's look at another problem that is similar but the events involved are not independent.

Problem: We have a bag of marbles with 4 yellow, 4 blue, and 3 green marbles. The marbles are also numbered so we can think of them as 1Y, 2Y, 3Y, 4Y, 1B, 2B, 3B, 4B, 1G, 2G, 3G. This can be thought of as the sample space of 11 equally likely outcomes. I will select a single marble from the bag. What is the probability that I pick a marble and it's the blue 2?

First, let's find the probability the intuitive way. There are 11 possibilities and 1 success. So the probability that the marble I pick is the blue 2 is $\frac{1}{11}$.

Now, let's use the rules. You can think of this as two events, "marble is blue" and "marble is a 2". Do you see the similarity between this and number 10?

Are these two events independent? To answer this question, find the following two probabilities. (Hint: Both can be found by thinking about the number of successes over the number of possibilities.)

P(marble is blue) =

P(marble is blue given it is a 2) =

Since these two probabilities are not equal, the two events are not independent. Use the formula **P(A and B) = P(A) * P(B given A)**. That is to say find
P(marble is blue and marble is 2) = P(marble is 2)P(marble is blue given marble is 2).

Did you get $\frac{1}{11}$, the answer gotten by finding the number of successes over the number of possibilities?

These two events were not independent because the bag of marbles lacked the symmetric nature of the deck of cards.