Math 128 Trigonometry
#42 section 6.4

Derivation of $\sin a - \sin b = 2\sin\left(\frac{a-b}{2}\right)\cos\left(\frac{a+b}{2}\right)$

$2\sin\left(\frac{a-b}{2}\right)\cos\left(\frac{a+b}{2}\right)$

$= 2\sin\left(\frac{a}{2}-\frac{b}{2}\right)\cos\left(\frac{a}{2}+\frac{b}{2}\right)$ (property of real numbers)

$= 2\left(\sin\frac{a}{2}\cos\frac{b}{2}-\cos\frac{a}{2}\sin\frac{b}{2}\right)\left(\cos\frac{a}{2}\cos\frac{b}{2}-\sin\frac{a}{2}\sin\frac{b}{2}\right)$ (pg 488 formulas: used the sine of a difference and the cosine of a sum)

$= 2\left(\sin\frac{a}{2}\cos\frac{a}{2}\cos^2\frac{b}{2}-\cos^2\frac{a}{2}\sin\frac{b}{2}\cos\frac{b}{2}-\sin^2\frac{a}{2}\sin\frac{b}{2}\cos\frac{b}{2}+\cos\frac{a}{2}\sin\frac{a}{2}\sin^2\frac{b}{2}\right)$

(FOILed the two binomials)

$= 2\left[\sin\frac{a}{2}\cos\frac{a}{2}\left(\cos^2\frac{b}{2}+\sin^2\frac{b}{2}\right)-\sin\frac{b}{2}\cos\frac{b}{2}\left(\cos^2\frac{a}{2}+\sin^2\frac{a}{2}\right)\right]$ (combining like terms)

$= 2\left[\sin\frac{a}{2}\cos\frac{a}{2}-\sin\frac{b}{2}\cos\frac{b}{2}\right]$ (Recall $\cos^2 q+\sin^2 q=1$ for any q.)

$= 2\left(\frac{\sin a}{2}-\frac{\sin b}{2}\right)$ (property pg 491: $\sin q\cos q=\frac{\sin 2q}{2}$. We used this twice.)

$= \sin a - \sin b$ (property of real numbers)

DONE!!!