Intermediate algebra NAME:
Practice Exam 1 Solutions
Sections 6.1 – 6.7

The actual exam will be 15 questions, with 12 show-your-work and 3 multiple choice questions. Your final exam (explained in syllabus) will be entirely multiple-choice.

Use the exam itself as scratch paper. Print your name at the top of your exam. Show work for possible partial credit. If an expression is prime, say so specifically.
The number in parentheses is the point value of the question. Circle your final answers. Good luck!

The difference of cubes formula is $a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)$**. The sum of cubes formula is** $a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$**. These formulas will be provided on the exam as well.**

1. (3) Find the GCF of the following three expressions. You do not need to factor it out; just give the GCF.

$50x^2yz^3, \quad 15xyz^5, \quad 20x^4y^2z^6$

$5xyz^3$

2. (3) Factor completely.

$3x^2 - 6x + 5x - 10$

$$3x^2 - 6x + 5x - 10$$
$$= 3x(x-2) + 5(x-2)$$
$$= (3x+5)(x-2)$$

3. (3) A friend of mine is trying to factor $5x^2 + 10x + 3x + 6$ by grouping. His work is below until he got stuck, at which time he quit. What is wrong with his work? Explain in words. Then fix the mistake and complete the factorization.

$$5x^2 + 10x + 3x + 6$$
$$= 5(x^2 + 2x) + 3(x+2)$$

He should have factored 5x, not just x, out of the first two terms. If he had done that, then what was left over from the first two terms would have matched what was left over from the last two terms and the next step could have been done.

$$5x^2 + 10x + 3x + 6$$
$$= 5x(x+2) + 3(x+2)$$
$$= (5x+3)(x+2)$$

4. (3) Factor completely.

$x^2 + 9x + 20$

$(x+4)(x+5)$

5. (3) Factor completely.

$4x^2 - 25$

Difference of two squares

$4x^2 - 25$

$= (2x)^2 - 5^2$

$= (2x+5)(2x-5)$

6. (3) Factor completely.

$x^3 - 27$

Difference of two cubes

$x^3 - 27$

$= x^3 - 3^3$

$= (x-3)(x^2 + x3 + 3^2)$

$= (x-3)(x^2 + 3x + 9)$

7. (3) Solve the following equation.

$(x+3)(2x+1) = 0$

$x+3=0$ *or* $2x+1=0$

$x=-3$ *or* $2x=-1$

$x = -1/2$

8. (3) Solve the following equation.

$3x^2 - 5x - 2 = 0$

$3x^2 - 5x - 2 = 0$

$(3x + 1)(x - 2) = 0$

$3x + 1 = 0$ *or* $x - 2 = 0$

$3x = -1$ *or* $x = 2$

$x = -1/3$

9. (3) Solve the following equation.

$x^2 + 6x + 8 = -1$

$x^2 + 6x + 8 = -1$

$x^2 + 6x + 9 = 0$

$(x + 3)(x + 3) = 0$

$x + 3 = 0$ *or* $x + 3 = 0$

$x = -3$ *or* $x = -3$

This is the same answer in both cases, so we only need to write it once. The solution is x = -3.

10. (3) The Zero Factor Property states that if the product of two numbers is zero, then one of the numbers must be zero. Explain, using words, how you use this fact to solve an equation like $x^2 + 14x + 45 = 0$.

We factor the left side so it is in the form of two things multiplied together. Then, because of the Zero Factor Property, we can separate this equation into two simpler-to-solve equations. We then solve each of those to get our final solutions.

$x^2 + 14x + 45 = 0$

$(x + 5)(x + 9) = 0$

$x + 5 = 0$ *or* $x + 9 = 0$

$x = -5$ *or* $x = -9$

11. (3) I solved the equation $4x^2 - 6x - 6 = 0$ and got two solutions, x = -1 and x = 3. Visibly check those answers in the original equation. Use proper notation and say specifically if my solutions are correct or not.

$4x^2 - 6x - 6 = 0$	$4x^2 - 6x - 6 = 0$
$4(-1)^2 - 6(-1) - 6 \stackrel{?}{=} 0$	$4(3)^2 - 6(3) - 6 \stackrel{?}{=} 0$
$4(1) + 6 - 6 \stackrel{?}{=} 0$	$4(9) - 18 - 6 \stackrel{?}{=} 0$
$4 + 6 - 6 \stackrel{?}{=} 0$	$36 - 18 - 6 \stackrel{?}{=} 0$
$4 \stackrel{?}{=} 0$	$12 \stackrel{?}{=} 0$
NO!	*NO*!

We substitute both "solutions" individually into the original equation to see if either makes it true. Since neither "solution" makes the original equation true, we say that both solutions are incorrect.

12. (3) Solve the following equation.
$x(x-2) = 24$

$x(x-2) = 24$
$x^2 - 2x = 24$
$x^2 - 2x - 24 = 0$
$(x-6)(x+4) = 0$
$x - 6 = 0$ *or* $x + 4 = 0$
$x = 6$ *or* $x = -4$

13. (5) The following question is similar to those on the worksheet "Story Problem Pieces" found on www.stlmath.com. Be sure to complete the table and answer the question that follows.

Consider two consecutive integers. Remember the set of integers is {…-3, -2, -1, 0, 1, 2, 3, …}. Complete the table below for the next consecutive integer.

First integer	Next consecutive integer
-2	***-1***
5	***6***
17	***18***

Now ask yourself "what did I do to the first integer to get the next consecutive integer?" So if I let x *represent the first integer, what is the next consecutive integer?*

I added one to the first number to get the second number. $x + 1$

14. (5) The following italicized problem is a story problem. You should follow the general plan for solving story problems outlined below. Points will be deducted if you do not define your variable, do not solve algebraically, or do not write your answer in phrase or sentence form. The verbal model is suggested but not mandatory.

There are two positive numbers whose product is 322. One of the numbers is five less than twice the other. Find the two numbers.

General plan:

a.) Decide which number you will let be *x*. Define your variable, letting *x* represent one of the numbers and then finding an expression involving *x* to represent the other number.

Let x = first number. So the second number would be 2x – 5.

b.) Draw a verbal model based on the general truth of the situation.

first number	*	*second number*	=	*product*

c.) Form an equation and solve it algebraically. Show your work. To help with the factoring stage, I have provided factors of -322 and -644 for your pleasure.

Both factors are given as positive, but remember to multiply to a negative number, one factor must be positive and the other negative.

Factors of -322
1, 2, 7, 14, 23, 46, 161, 322

Factors of -644
1, 2, 4, 7, 14, 23, 28, 46, 92, 161, 322, 644

$x(2x-5)=322$

$2x^2-5x=322$

$2x^2-5x-322=0$

$2x^2-28x+23x-322=0$

$2x(x-14)+23(x-14)=0$

$(2x+23)(x-14)=0$

$2x+23=0$ *or* $x-14=0$

$2x=-23$ *or* $x=14$

$x=-23/2$

Here, the first number could be -23/2 or 14. But since the problem stated the numbers are positive, we know the first number must be 14. So we then calculate 2(14) – 5 to find the second number to be 23.

d.) Write your answer in sentence or phrase form. Make sure you tell me both numbers.

The two numbers are 14 and 23.

The following questions are multiple-choice. Please write the letter of your choice in the blank provided.

15. (3) Which of the following is the GCF of the expressions below? 15 _____A___
$50x^2y^5z, \quad 70xyz^3, \quad 30x^4yz$

a.) $10xyz$
b.) $150x^4y^5z^3$
c.) $10x^2y$
d.) $150x^7y^7z^5$

16. (3) Completely factor the following. 16 _____B___
$18ab^2 + 30a^2b^2 - 6a$

a.) $6a$
b.) $6a\left(3b^2 + 5ab^2 - 1\right)$
c.) $6a\left(12b^2 + 24ab^2 - 0\right)$
d.) $3a\left(6b^2 + 10ab^2 - 2\right)$

17. (3) Completely factor the following trinomial. 17 ____C____
$8x^2 - 10x - 3$

a.) $(8x-3)(x+1)$
b.) $(4x-3)(2x+1)$
c.) $(4x+1)(2x-3)$
d.) prime: cannot be factored