Intermediate Algebra NAME:
Practice Exam 2 Solutions
Section 7.1 – 7.6, 9.2

Use the test itself as scratch paper. Print your name at the top of your test. Show work for possible partial credit. No expressions that you are asked to factor will be prime. The symbols $\cdot$ and $*$ both indicate multiplication. The number in parentheses is the point value of the question.

Leave your answers as reduced fractions or write them as decimal answers, rounded to two decimal places. (This means if your answer is left in decimal form, there should be two digits to the right of the decimal point.) If an answer turns out to be undefined, say so specifically. Circle your final answers. Good luck!

1. (3) We say that the fraction $\frac{4}{12}$ is equivalent to $\frac{1}{3}$. Explain this using the fact that $\frac{4}{12}$ can be broken down as $\frac{1*4}{3*4}$. Explain **specifically why** the 4's can be left out to write $\frac{4}{12}$ as simply $\frac{1}{3}$.

We can think of $\frac{1*4}{3*4}$ *as* $\frac{1}{3}*\frac{4}{4}$. *But* $\frac{4}{4}$ *is just another name for 1. So it's really like* $\frac{1}{3}$ *times 1 which makes* $\frac{1}{3}$.

2. (4) Let $r(x)=\frac{3x+9}{x^2+2x+1}$. Find $r(3)$.

$$r(3)=\frac{3(3)+9}{(3)^2+2(3)+1}=\frac{9+9}{9+6+1}=1.125$$

If you put this into your calculator, be sure to put the entire top in parentheses and the entire bottom in parentheses. It should look like $(3*3+9)/(3^2+2*3+1)$.

3. (3) Perform the indicated operation and simplify.

$$\frac{x^2}{x-4}-\frac{x+12}{x-4}=\frac{x^2-x-12}{x-4}=\frac{(x-4)(x+3)}{x-4}=x+3$$

4. (3) Verify that the expressions $2x-5$ and $5-2x$ are opposites. In other words, choose any value for x and substitute it into these two different expressions. Simplify each so that it is obvious that they are opposites.

I will substitute 3 in for x. I chose that value arbitrarily. It is not good to choose 1 or 0.

$2x-5=2*3-5=6-5=1$

$5-2x=5-2*3=5-6=-1$

5. (4) Simplify the following.

$$\frac{x^2-7x+12}{\left(9-x^2\right)(x+5)}=\frac{(x-4)(x-3)}{(3-x)(3+x)(x+5)}$$

$$=\frac{-1(x-4)\cancel{(x-3)}}{\cancel{(3-x)}(3+x)(x+5)}$$

$$=\frac{-1(x-4)}{(3+x)(x+5)}$$

Notice we have a difference of squares on bottom so we factor it out too.
We cancel x – 3 on top with 3 – x on bottom, we are left with a factor of -1. I wrote it on top in front so it would not get lost.

6. (3) Perform the indicated operation and simplify.

$$\frac{x^2+2x-24}{x^2-x-12}\cdot\frac{x^2+5x+6}{x^2+8x+12}=\frac{(x+6)(x-4)(x+3)(x+2)}{(x-4)(x+3)(x+6)(x+2)}$$

$$=\frac{\cancel{(x+6)}\cancel{(x-4)}\cancel{(x+3)}\cancel{(x+2)}}{\cancel{(x-4)}\cancel{(x+3)}\cancel{(x+6)}\cancel{(x+2)}}$$

$$=1$$

Here, everything cancels out and we are left with factors of 1 on top and bottom, which reduces to 1.

7. (3) Perform the operation indicated and simplify.

$$\frac{4}{x+2}-\frac{5}{x-4}=\frac{4}{x+2}\frac{(x-4)}{(x-4)}-\frac{5}{x-4}\frac{(x+2)}{(x+2)}$$

$$=\frac{4(x-4)}{(x-4)(x+2)}-\frac{5(x+2)}{(x-4)(x+2)}$$

$$=\frac{4(x-4)-5(x+2)}{(x-4)(x+2)}$$

$$=\frac{4x-16-5x-10}{(x-4)(x+2)}$$

$$=\frac{-1x-26}{(x-4)(x+2)}$$

8. (3) Marie can mow the lawn in 5 hours. Joe can do the same job in 7 hours. If they work together, how long will it take the two of them to mow the lawn? Be sure to define your variable, solve the problem using algebra, and include units in your answer.

Let x represent the number of hours it takes them to mow the lawn together. The next thing to do is to write out a verbal model. Think about the situation and what is true. Below I have written my verbal model.

Part of lawn Marie mows	+	Part of lawn Joe mows	=	1 whole lawn

Now, how much of the lawn does Marie mow? Well, she mows the whole lawn in 5 hours, so she should be able to mow $^1/_5$ *of the lawn in one hour. And she will mow for x hours. So, total, she should end up mowing* $^1/_5$ *times x or* $^x/_5$ *of the lawn. Likewise, Joe will mow* $^x/_7$ *of the lawn. I form my equation, following my verbal model.*

$$\frac{x}{5}+\frac{x}{7}=1$$
$$7x+5x=35$$
$$12x=35$$
$$x=2.92$$

I start by multiplying every term by the LCD, 35. That simplifies the equation a lot by getting rid of the fractions. Then I combine like terms on the left and finally divide by 12 to get x alone. Since x was the number of hours it would take them to mow the lawn together, and x is 2.92, it should take them 2.92 hours to mow the lawn. Notice the units (hours) is an important part of the answer.

9. (3) Solve the following equation. Check your answers and be sure to cross out any that turn out to be wrong.

$$\frac{3}{x-7}=\frac{7}{x+5}$$
$$3(x+5)=7(x-7)$$
$$3x+15=7x-49$$
$$64=4x$$
$$16=x$$
$$x=16$$

I started by cross-multiplying. (That will only work if your equation is "one fraction equals one fraction". It would not work if there were two fractions on one side.) That eliminates the fractions which makes me very happy. Then I simplify each side by distributing the 3 and 7. Now we've got a linear equation so I get the x's on one side and the constants on the other. I chose to add 49 to and subtract 3x from both sides so that the coefficients I was left with would be positive. I then divide by 4 to get 16. I rewrote it as x = 16 because it just looks nicer.

10. (3) Solve the following equation. Check your answers and be sure to cross out any that turn out to be wrong.

$$\frac{4}{x+4}+\frac{3}{x-4}=\frac{24}{x^2-16}$$
$$4(x-4)+3(x+4)=24$$
$$4x-16+3x+12=24$$
$$7x-4=24$$
$$7x=28$$
$$x=4$$

First, we have to see that x^2-16 is really $(x-4)(x+4)$. So the LCD is this $(x-4)(x+4)$. We multiply everything by that to eliminate the fractions. This creates a linear equation we then solve. Distribute on the left and combine like terms. Add 4 and divide by 7 to get an initial solution of 4. BUT… Does 4 really work in the equation? Like every equation I solve, especially rational equations, I should check the answer in the original equation. When we do that, we see that 4 actually makes the second and third fractions undefined. So we have to disregard our solution. So there is ***no solution****.*

11. (3) Solve the following equation.

$$|4x+2|-5=10$$
$$|4x+2|=15$$
$$4x+2=15 \quad or \quad 4x+2=-15$$
$$4x=13 \quad or \quad 4x=-17$$
$$x=13/4 \quad or \quad x=-17/4$$

First, isolate the absolute value part so you can perform the procedure for absolute value equations. Once we get the absolute value part isolated, we can think "if the absolute value of some number is 15, then that number must be 15 or -15". (Here, the "number" is "4x + 2".) That leads us to form the two linear equations. Solve them to get the two solutions. You should always check them but you will see that they do work.

12. (3) If two numbers have the same absolute value, what must be true of the numbers? In other words, if we say $|x|=|y|$, what can we say about *x* and *y*? (Hint: There are two possibilities here.)

The numbers are either the same such as $|5|=|5|$ *or opposites of each other such as* $|5|=|-5|$.

These are multiple-choice questions. Write the letter of your choice in the blank provided.

13. (3) Simplify. 13. ____B___

$$\frac{x^2 - 6x + 5}{2x^2 + x - 3}$$

a.) $\frac{3}{2+x}$

b.) $\frac{x-5}{2x+3}$

c.) $\frac{x-5}{2x-3}$

d.) $\frac{x+6}{2x+3}$

e.) $\frac{0}{0}$

14. (3) What makes a rational expression or a fraction "undefined"? 14. ___A____

a.) The denominator has a value of zero.
b.) The numerator has a value of zero.
c.) Not being able to add it to another rational expression or fraction.
d.) Not being able to find its reciprocal.
e.) Not being able to find a least common denominator.

15. (3) Solve the following equation. 15. ___E____

$2|5x-3|+7=21$

a.) $x = 2$

b.) $x = 17/5$

c.) $x = 17/5$ or $x = -17/5$

d.) $x = 2$ or $x = 4/5$

e.) $x = 2$ or $x = -4/5$