Intermediate Algebra NAME:
Practice Exam 3 Solutions
Sections 10.1 – 10.6

Use the test itself as scratch paper. Print your name at the top of your test. Show work for possible partial credit. The number in parentheses is the point value of the question.

Circle your final answers. Good luck! The rules of exponents, given below, are true for real numbers a, b, m, and n.

$a^m * a^n = a^{m+n}$ $\left(a^m\right)^n = a^{mn}$

$(ab)^n = a^n b^n$ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$

$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$ $a^0 = 1, a \neq 0$

$a^{-n} = \frac{1}{a^n}, a \neq 0$

The distance between two points (x_1, y_1) **and** (x_2, y_2) **is given by** $d = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$ **.** **The midpoint of the segment between two points** (x_1, y_1) **and** (x_2, y_2) **is the point with coordinates** $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$**.**

1. (6) Find the following on your calculator. Write your answers as decimals, rounded to two decimal places.

a.) $\sqrt{48} \approx 6.93$ *On the TI calculators, you use the second function of the "x squared" key along the left of the calculator.*

b.) $\sqrt[4]{4096} = 8$ *On the TI calculators, you need to enter a "4" onto the home screen. Then press MATH and select 5. It should look like* $^x\sqrt{\ }$ *. Then enter the 4096 and press ENTER. Notice* 8^4 *is in fact 4096.*

c.) $\sqrt[3]{99} \approx 4.63$ *You can do this like I did part b, but since cube roots are used frequently, the calculator has this function built in. Press MATH and select 4 for* $\sqrt[3]{\ }($*. Then put the 99 in and press ENTER. Notice if you use this function, as opposed to 5 in part b, you do not need to enter the 3 first.*

2. (3) Simplify the following. Assume all variables represent non-negative numbers.

$$\sqrt{64x^8y^2}$$
$$= \sqrt{64}\sqrt{x^8}\sqrt{y^2}$$
$$= 8x^4y$$

Here, think of each factor on its own and find its square root. First, the square root of 64 is 8 because 8 squared is 64. We deal with the x^8 and y^2 the same way. What squared would make x^8 ? Well, because of the rules of exponents, $\left(x^4\right)^2 = x^8$, right? So the answer is x^4. Likewise, the thing that I square to get y^2 would be simply y. Put these all together and get $8x^4y$ as the final answer.

3. (3) Let $f(x) = \sqrt{5x+6} - 2$. Find $f(6)$.

$$f(6) = \sqrt{5*6+6} - 2$$
$$= \sqrt{30+6} - 2$$
$$= \sqrt{36} - 2$$
$$= 6 - 2$$
$$= 4$$

Put 6 into the formula for x and simplify. You need to follow the order of operations. So we see that f(6) = 4. Recall that f(x) is just another name for y. So this is another way to say that for this function, when x is 6, y is 4.

4. (3) Simplify the following. (If it helps to write it as a radical expression first, then do so. However, I will not deduct points for leaving this step out.)

$$\left(125x^6y^3\right)^{1/3}$$
$$= \sqrt[3]{125x^6y^3}$$
$$= \sqrt[3]{125}\sqrt[3]{x^6}\sqrt[3]{y^3}$$
$$= 5x^2y$$

This is done similarly to #2 but instead of thinking about what we would square to get the numbers inside the radical, we are thinking what we would cube to get the numbers inside. I still break it up into separate cube roots (line 3), even if I do not write that out.

5. (3) Simplify the following using the rules of exponents. Leave your answer with fractional exponents.

$$y^{1/2}y^{1/3} = y^{1/2+1/3} = y^{3/6+2/6} = y^{5/6}$$

Use the rules of exponents to see that we need to add the exponents. But that means adding two fractions with unlike denominators, so we need to get like denominators. We do that and then add. If needed, we would simplify that final exponent, but it is already in reduced form so we are done.

6. (3) Rewrite the following with positive exponents. Then simplify your answer. Leave your answer as a fraction.

$$36^{-1/2} = \frac{1}{36^{1/2}} = \frac{1}{\sqrt{36}} = \frac{1}{6}$$

You have to remember what a negative exponent does. A number raised to a negative power is one over that number raised to the positive power. Then it is a matter of realizing that $36^{1/2}$ is simply 6. I wrote it in radical form so it is more obvious. So we end up with 1/6.

7. (3) Solve the following equation. Be sure to check your answer(s).

$$\sqrt{x+4} = 5$$
$$\left(\sqrt{x+4}\right)^2 = 5^2$$
$$x+4 = 25$$
$$x = 21$$

*Square both sides to get rid of the square root. Then subtract 4 to get of the "plus 4" on the left. This leaves us with x is 21. You should always check your answers by plugging them back into the **original** equation to make sure they work.*

8. (3) Solve the following equation. Be sure to check your answer(s) and cross out any that turn out to be wrong.

$$x = \sqrt{6x-8}$$
$$x^2 = \left(\sqrt{6x-8}\right)^2$$
$$x^2 = 6x-8$$
$$x^2 - 6x + 8 = 0$$
$$(x-4)(x-2) = 0$$
$$x-4=0 \quad or \quad x-2=0$$
$$x=4 \quad or \quad x=2$$

This starts off with the same process as above but it turns into a quadratic equation when we square both sides. Solving that means to get 0 on one side, factor the other side, and set each factor to 0. This creates two linear equations we solve to find two potential answers. But, once again, we need to check them both.
Since $4 = \sqrt{6*4-8}$ and $2 = \sqrt{6*2-8}$ both check out, we see that both 4 and 2 are valid solutions. You should definitely simplify each to see this for yourself.

9. (3) What is the difference between $\sqrt{-36}$ and $-\sqrt{36}$? (Notice where the negative sign is and think about how this affects it.)

The $\sqrt{-36}$ *is the number that I square to get -36. But there is no such real number. So we say it does not exist in the real numbers. The answer turns out to be a complex number which the book finds but we will simply say it does not exist in the real numbers. You will explore complex numbers in College Algebra. On the other hand,* $-\sqrt{36}$ *is the negative of the square root of 36. Since the square root of 36 is 6, we see that* $-\sqrt{36}$ *is equal to -6.*

10. (3) Completely simplify the following. Assume all variables represent non-negative numbers.

$$\frac{\sqrt{180x^4}}{6x}$$
$$=\frac{\sqrt{36*5\left(x^2\right)^2}}{6x}$$
$$=\frac{\sqrt{36}\sqrt{5}\sqrt{\left(x^2\right)^2}}{6x}$$
$$=\frac{6\sqrt{5}\left(x^2\right)}{6x}$$
$$=x\sqrt{5}$$

Look for the largest perfect square that is a factor of 180. The idea is to write 180 as 36 times 5, and then take the 36 out of the square root symbol (radical) as simply 6, leaving the 5 under the radical. You also want to notice how x^4 *could be thought of as* x^2*, squared. So we can reduce "the square root of* x^4*" as* x^2*.*

Once we get as much out of the radical as we can (second line from bottom), then cancel the 6 and x from top and bottom of the fraction to get the final answer. Remember you can only cancel things that are factors on top and bottom of a fraction.

11. (3) Algebraically find the midpoint of the segment whose endpoints are (5, -10) and (0, 6). Show your work.

(x_1, y_1) and (x_2, y_2)

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$
$$=\left(\frac{5+0}{2},\frac{-10+6}{2}\right)$$
$$=\left(\frac{5}{2},\frac{-4}{2}\right)$$
$$=\left(2.5,-2\right)$$

The first step is to recognize what formula to use and that the two points given are (x_1, y_1) *and* (x_2, y_2)*. Notice the formula basically averages the x values and averages the y values to come up with a new point, with an x coordinate and a y coordinate. We substitute the x and y values into the formula and simplify. Again, notice you end up with an ordered pair, since the midpoint is a point, with an x value and a y value.*

12. (3) Perform the indicated operation and completely simplify the following. Assume all variables represent non-negative numbers.

$$\left(\sqrt{4x}+7\right)\left(\sqrt{x}-5\right)$$

F O I L

$$=\sqrt{4x}\sqrt{x}-5\sqrt{4x}+7\sqrt{x}-7*5$$
$$=\sqrt{4x^2}-5\sqrt{4x}+7\sqrt{x}-35$$
$$=2x-5*2\sqrt{x}+7\sqrt{x}-35$$
$$=2x-10\sqrt{x}+7\sqrt{x}-35$$
$$=2x-3\sqrt{x}-35$$

This is just like a multiplication problem like (x + 2)(x + 3). We start off by FOILing the stuff out. Once you do that, simplify each term and then combine like terms.

Remember that you can simplify square root terms by taking out the perfect squares. For instance, $\sqrt{4x}$ *can be written as* $2\sqrt{x}$ *because* $\sqrt{4}$ *is another name for 2.*

These are multiple-choice questions. Write the letter of your choice in the blank provided.

13. (3) Perform the indicated operation and completely simplify the following. Assume all variables represent non-negative numbers.

13. ____C___

$$\sqrt{\frac{28}{x^2}}+\sqrt{\frac{7}{4x^2}}$$

a.) no solution

b.) $\frac{\sqrt{35}}{2x^2}$

c.) $\frac{5\sqrt{7}}{2x}$

d.) $\frac{119}{4x^2}$

e.) $\frac{\sqrt{7}}{x}$

$$\sqrt{\frac{28}{x^2}}+\sqrt{\frac{7}{4x^2}}$$
$$=\frac{\sqrt{28}}{\sqrt{x^2}}+\frac{\sqrt{7}}{\sqrt{4x^2}}$$
$$=\frac{2\sqrt{7}}{x}+\frac{\sqrt{7}}{2x}$$
$$=\frac{2}{2}\frac{2\sqrt{7}}{x}+\frac{\sqrt{7}}{2x}$$
$$=\frac{4\sqrt{7}}{2x}+\frac{\sqrt{7}}{2x}$$
$$=\frac{5\sqrt{7}}{2x}$$

I like to think of this as a normal, add-two-fractions problem. I need to get like denominators and then add. So I simplify each fraction, noticing how the denominators can be taken out of the radicals. On the third line, I see that my denominators are x and 2x; I need to get like denominators. The first fraction needs a factor of 2 to make the denominators the same. So multiply top and bottom of that fraction by 2, getting us to the fifth line. Add the tops and keep the bottom, just like adding any two fractions with the same denominators. You can think of $4\sqrt{7}+\sqrt{7}$ *the same as you would 4x + x, and simply add like terms.*

14. (3) Rationalize the denominator. Completely simplify your final answer. 14. ___A____

$$\frac{\sqrt{5x^2}}{\sqrt{7}}$$

a.) $\frac{\sqrt{35}x}{7}$

b.) $\frac{5x}{1}$

c.) $\frac{5x^2}{7}$

d.) $\frac{\sqrt{5}x}{\sqrt{7}}$

e.) $\frac{5x}{\sqrt{35}}$

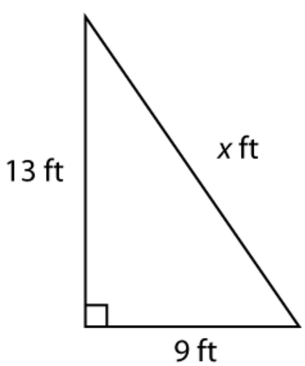
$$\frac{\sqrt{5x^2}}{\sqrt{7}}\frac{\sqrt{7}}{\sqrt{7}}$$

$$=\frac{\sqrt{35x^2}}{\sqrt{49}}$$

$$=\frac{x\sqrt{35}}{7}$$

Rationalizing a denominator means to eliminate the radical from the bottom while not changing the value of the overall fraction. Multiply both top and bottom by $\sqrt{7}$ *. This makes the square root of 49 or simply 7 on bottom. Multiplying on both the top and bottom means we did not really change the value of the fraction; we just changed its form. Notice the top then reduces because of the square root of x squared. You can leave your answer as I did here or write it as answer A is written. It is sometimes safer to write it with the x in front, so it does not accidentally get buried inside the radical, especially if your writing tends to be quick and messy.*

15. (3) Find the length of the hypotenuse for the right triangle shown below. Convert your answer to decimal form, rounded to two decimal places. 15. ___B____



a.) 22 ft
b.) 15.81 ft
c.) 6.61 ft
d.) 484 ft
e.) 9.38 ft

Use the Pythagorean Theorem.
Simplify the right. Once you get 250 on the right, take the square root of both sides to get x alone. I always round to two decimal places. Make sure to include the proper units, feet.

$$x^2 = 13^2 + 9^2$$

$$x^2 = 169 + 81$$

$$x^2 = 250$$

$$\sqrt{x^2} = \sqrt{250}$$

$$x \approx 15.81 \textit{ feet}$$