Binary Magic Trick (Section 2.3)   NAMES:

The number trick shown in class goes as follows.

Ask a person to secretly think of a number between 1 and 60. You will be able to "guess" their number by asking them on which of the six cards below the number appears. We will investigate how the trick is done.

1	3	5	7	9	11
13	15	17	19	21	23
25	27	29	31	33	35
37	39	41	43	45	47
49	51	53	55	57	59

Card 1

8	9	10	11	12	13
14	15	24	25	26	27
28	29	30	31	40	41
42	43	44	45	46	47
56	57	58	59	60	

Card 4

2	3	6	7	10	11
14	15	18	19	22	23
26	27	30	31	34	35
38	39	42	43	46	47
50	51	54	55	58	59

Card 2

16	17	18	19	20	21
22	23	24	25	26	27
28	29	30	31	48	49
50	51	52	53	54	55
56	57	58	59	60	

Card 5

4	5	6	7	12	13
14	15	20	21	22	23
28	29	30	31	36	37
38	39	44	45	46	47
52	53	54	55	60	

Card 3

32	33	34	35	36	37
38	39	40	41	42	43
44	45	46	47	48	49
50	51	52	53	54	55
56	57	58	59	60	

Card 6

This trick is based on the binary sequence, in particular how the numbers 1 through 60 are written in binary. I have written those numbers in binary on the next page.

We will first investigate how the binary system works and then see how the trick itself works.

Decimal to Binary Conversion
Numbers 1 through 60

1	000001	21	010101	41	101001
2	000010	22	010110	42	101010
3	000011	23	010111	43	101011
4	000100	24	011000	44	101100
5	000101	25	011001	45	101101
6	000110	26	011010	46	101110
7	000111	27	011011	47	101111
8	001000	28	011100	48	110000
9	001001	29	011101	49	110001
10	001010	30	011110	50	110010
11	001011	31	011111	51	110011
12	001100	32	100000	52	110100
13	001101	33	100001	53	110101
14	001110	34	100010	54	110110
15	001111	35	100011	55	110111
16	010000	36	100100	56	111000
17	010001	37	100101	57	111001
18	010010	38	100110	58	111010
19	010011	39	100111	59	111011
20	010100	40	101000	60	111100

1. Make sense of the binary numbers. Recall, from our reading, that a decimal number (our normal system of writing numbers) is converted to binary by finding how we would add 1, 2, 4, 8, 16, 32, 64, etc. to get the decimal number. Use this to explain how 13 is written as 001101 in binary.

The cards on page 1 were devised in the following manner.
Card 1 is composed of the decimal numbers that had a 1 in the far right column of their binary version (meaning had 1 in their sum, as found in question 1 above).
Card 2 is composed of the decimal numbers that had a 1 in the second-to-right column of their binary version (meaning had 2 in their sum, as found in question 1 above).
Card 3 is composed of the decimal numbers that had a 1 in the third-to-right column of their binary version (meaning had 4 in their sum, as found in question 1 above).
Card 4 is composed of the decimal numbers that had a 1 in the fourth-to-right column of their binary version (meaning had 8 in their sum, as found in question 1 above).
Cards 5 and 6 continued this pattern.

3. Let's work through a sample number as if doing the trick. Let's say our dupe picks the number 43. On which cards does this number appear?

4a. If the number 43 appears on Card 1, then what must be part of its sum?

4b. If the number 43 appears on Card 2, then what must be part of its sum?

4c. If the number 43 appears on Card 4, then what must be part of its sum?

4d. If the number 43 appears on Card 6, then what must be part of its sum?

4e. The number did not appear on cards 3 and 5. What powers of 2 are not, therefore, part of its sum?

4f. Add the values in questions 4a through 4d to see that they add up to 43.

5. Try the trick out with another number. Pick a "secret" number between 1 and 60. Write which cards your "secret" number appears on and how to use that to find the "secret" number.