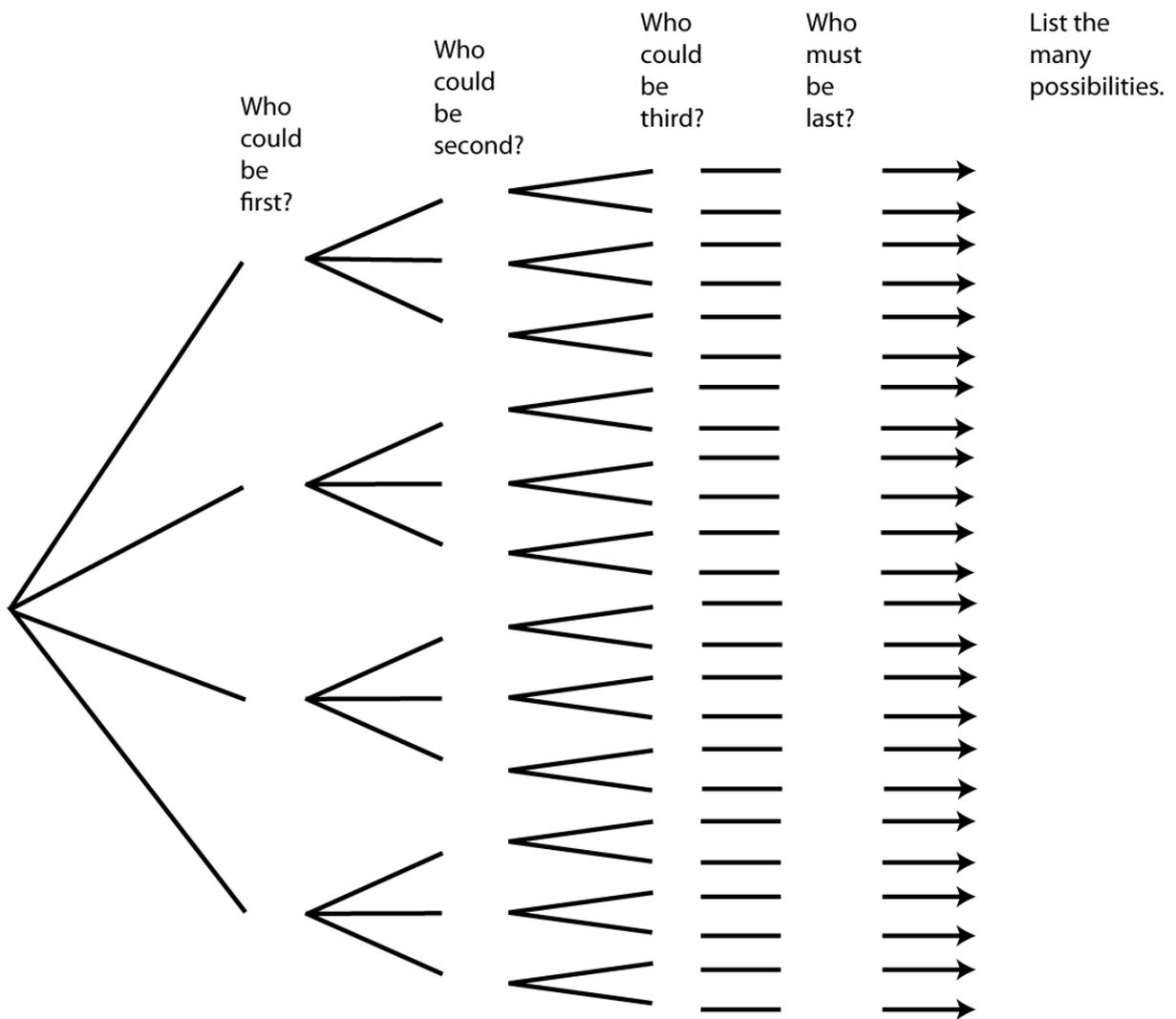


Abby, Bob, Cathy, and Doug are lining up outside of a theatre to buy tickets to a show. How many ways can this happen? We will investigate the many possibilities for the order of this line.

1. Write down one possible order for these four people to buy tickets.

2. Fill in the tree diagram to draw out and organize the possibilities for the 1st, 2nd, 3rd, and 4th places in line at the theatre. The first branching should represent the first person in line; the second branching should represent the second person in line, and so on.



3. Use your tree diagram to write out the many possibilities (of all four places) to the far right. How many possibilities are there?

4. The Fundamental Counting Principle tells us how many possibilities we should have. It states that if you have a task (like lining people up outside a theatre) that takes k parts (like k places in line), and the first part can be done in n_1 ways, the second part can be done in n_2 ways, ... the k^{th} part can be done in n_k ways, then the number of ways you can do the whole task is $n_1 * n_2 * n_3 * \dots * n_k$.

Now, in our problem, k is 4, the number of places in line. When you place the first person in line, there are 4 people to choose from. When you place the second person in line, there are only 3 people to choose from, since the first person is already standing in line. When you place the third person, there are 2 people to choose from. When you place the fourth person in line, there is only 1 person to choose from.

Show that the Fundamental Counting Principle finds the number of possibilities you found in question 3.

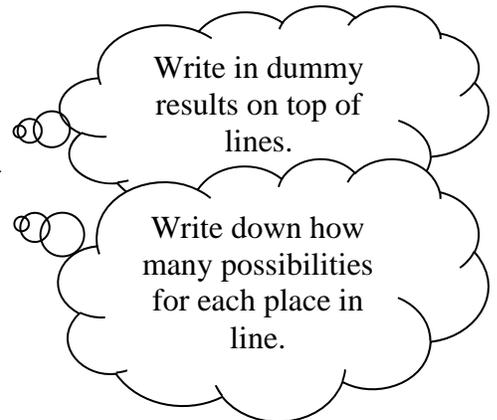
Notice this answer could be thought of as $4!$ (“four factorial”).

Factorials are a quick way to write the product of any integer and all the positive integers less than it. For instance, $10! = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$.

5. Let’s mix up the problem a bit. What if Edgar and Fred come by and stand in line with them? How many ways can the six of them line up? Let’s think about it together.

A good way to think through these problems is to draw spaces for each part of the task (like the six places in line) and ask yourself how many ways can each be filled. As you go, fill in a dummy name. Practice it below.

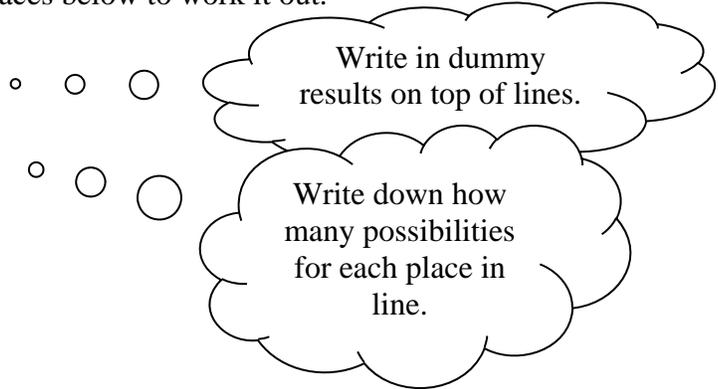
1st in line 2nd in line 3rd in line 4th in line 5th in line 6th in line



So, how many ways can the six of them line up?
Write your final answer in factorial form too.

6. Suppose only the first four places will be able to buy tickets. If the six people (Abby, Bob, Cathy, Doug, Edgar, and Fred) show up, how many ways can four of them line up in the first four places in line? Use the spaces below to work it out.

1st in line
2nd in line
3rd in line
4th in line



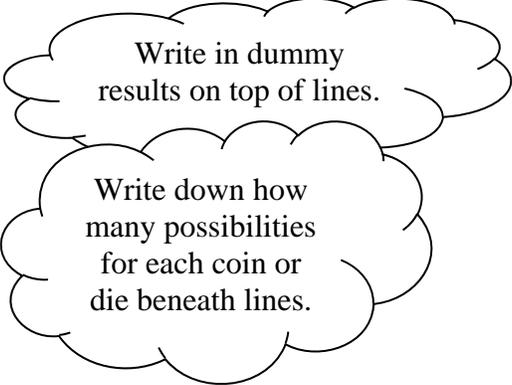
7a. A menu is reproduced below. A special is underway where a customer can order one appetizer, one entree, and one dessert for the low, low price of \$12.95. How many different ways can this be done?

| Appetizers | Entrees | Desserts |
|-------------------|---------------------|-----------------|
| Clam sampler | Fried Calamari | Chocolate cake |
| Shrimp cocktail | Lemon Zested Salmon | Cherry pie |
| Mozzarella sticks | Curried Chicken | |
| | Beef a la Teriyaki | |
| | Vegetarian Delight | |

7b. To get an idea of what we are counting, write out just three of the many possibilities.

8a. Three coins (a nickel, a dime, and a penny) and a fair, six-sided die are tossed into the air. We will record whether a heads or tails shows on each coin and the number on the die. (For instance, HHT5 could be a result.) How many different results are possible? Use the spaces below and the Fundamental Counting Principle to help you.

| | | | |
|--------|------|-------|-----|
| | | | |
| nickel | dime | penny | die |



Write in dummy results on top of lines.

Write down how many possibilities for each coin or die beneath lines.

8b. These 48 possibilities are a little daunting to write out, so let's consider just the nickel, dime, and penny for now. How many ways can these three coins turn out? (Ignore the die for now.) Make a tree diagram and then list out these possibilities.

8c. In addition to the coins, let's consider the die now. Consider the results from question 8b but add on the die result at the end. Explain how this would make the 48 possibilities we see in question 8a. You are not required to write all of these 48 results down but do explain their pattern.

9a. An algebra test has four multiple-choice questions with five answers (A, B, C, D, E) each, followed by three true-false questions. How many answer keys are possible? (A teacher devises one possible answer key when she writes a test. I want to know how many possibilities she has.)

9b. Below are seven blank answer spaces. Fill in a possible answer key for the test described above. Use it to explain your answer to question 9a.

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

10a. Ten people are competing in a dance competition. How many ways could the top three positions be chosen?

10b. Let's say these ten people are abbreviated as contestants A, B, C, D, E, F, G, H, I, and J. Write down six different ways that three of these people can stand on the winner's podium. Use them to explain your answer to part a.

11a. Draw out a tree diagram for the following situation to form a systematic list of all possible answer keys.

A biology quiz has two true/false questions, followed by one multiple-choice question that has four possible answers.

11b. List out every possible answer key for this quiz. If you did this neatly above, you do not need to rewrite your list.