Inductive Versus Deductive Reasoning NAME:

There are two major types of reasoning that we use in math and science. These are **inductive reasoning** and **deductive reasoning**.

Inductive reasoning looks for patterns or examples that share some common characteristic. From the pattern, we conclude some general statement.

For instance, look at the string of numbers 2, 4, 8, 16, …

What would you guess the next number is?

When you use a set of examples or a pattern to make a conjecture, you are using inductive reasoning. This is often described as "going from the specific to the general".

Deductive reasoning essentially goes the opposite way. It is often described as "going from the general to the specific".

For instance, we know general truths like the properties of real numbers and you can add a number to both sides of an equation and it remains a true equation. Right? Deductive reasoning allows us to be sure of the conclusion when solving an equation like below.

$$4(x+2)=24$$
$$4x+8=24$$
$$4x=16$$
$$x=4$$

> The distribution property simplifies the left side. Then we subtract 8 from both sides and divide both sides by 4. **All of these steps are true for numbers in general.**

We use deductive reasoning when we want to prove, without a doubt, that our conclusion is true. Inductive reasoning arguments can give us a reasonable conclusion but cannot guarantee that the conclusion is, in fact, the truth. For instance, perhaps the string of numbers from above (2, 4, 8, 16, …) were the ages of the people in my household so that the next number is 45?

It may seem underhanded to say that (up above, I was thinking of the number 32 as the next number, were you?) but it points out that inductive reasoning may get you a correct conclusion but you never really *know* for sure. There is not enough evidence to say absolutely that the pattern you saw was the one the writer had intended. There may be other valid conclusions as well.

Let’s investigate inductive versus deductive reasoning a bit further with this problem.

Problem: A 4-digit palindrome is a 4-digit number that is the same backwards as forwards, like 5775. Is every 4-digit palindrome divisible by 11?

Let’s start off by trying to understand divisibility. What does it mean for a number to be divisible by 11? Give an example of such a number (*not* necessarily a four digit one) and tell what is special about it.

Inductive reasoning: Think up at least four 4-digit palindromes and see if they are divisible by 11. Write your examples below.

4-digit palindrome ____________ Is it divisible by 11?__________

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4-digit palindrome ____________ Is it divisible by 11?__________

Using inductive reasoning (that is, look at your examples), would you conclude that *every* 4-digit palindrome is divisible by 11?

Some preliminaries:

1. A number is divisible by 11 if you can write it as a whole number (0, 1, 2, 3, …) times 11. Give an example of this.

2. The distribution property of real numbers is quite handy. I like it because it shows how we could take a common factor out of two things we are adding (called terms) to rewrite the expression as a multiplication problem. Can you think of an example of this property?

Deductive reasoning: Now, let's attempt to look at this 4-digit palindrome problem in general. It is *not* sufficient to just look at examples. Using letters for the digits, let's write a generic 4-digit palindrome as "*abba*". (Here, a and b represent digits taken from 1 through 9. Technically, b could be 0.) Follow the steps below for the proof.

3. Now, the easiest way for us to get to our conclusion is to explore this 4-digit palindrome in extended form. For instance, the number 5775 could be written as

$$5775 = 5 \cdot 1000 + 7 \cdot 100 + 7 \cdot 10 + 5 \cdot 1.$$

Can you write $abba$ in extended form?

4. Combine like terms in your answer to simplify it.

5. How does the distribution property allow us to rewrite this expression? In other words, do you see any way to "factor out" a common factor of the two terms you should now have? What is that common factor?

6. Another super cool property of real numbers is closure. We say that whole numbers are closed under multiplication and addition. What that means is if you multiply or add two whole numbers, you will get a whole number as the result. Always. Do you believe me? Try to think of two whole numbers whose sum is *not* a whole number? Are you convinced?

So, considering the closure of whole numbers, what can be said about our expression we left back in question 5? In particular, look at the part that should be in parentheses after using the distribution property.

7. To wrap this up, we see that our generic 4-digit palindrome is the product of 11 and a whole number. Can we conclude that it is necessarily divisible by 11? In other words, did we prove what we intended?

Inductive reasoning can help us form a good hypothesis. Deductive reasoning provides the undeniable proof.